BULLETIN
DE
L'ACADÉMIE POLONAISE DES SCIENCES

SÉRIE DES SCIENCES
MATHÉMATIQUES, ASTRONOMIQUES ET PHYSIQUES

Volume VIII Numéro 2

VARSOVIE 1960

Le Bulletin de l'Académie Polonaise des Sciences est publié en Séries.

La présente Série comprend des communications du domaine de la mathématique, astronomie et physique

Lesdites sciences sont du domaine de la Troisième Classe de l'Académie Polonaise des Sciences.

La présente Série paraît mensuellement.

Les communications destinées à être publiées dans le Bulletin sont présentées à la Rédaction du Bulletin par les membres de l'Académie.

Adresse de la Rédaction: Pałac Kultury i Nauki, Varsovie

BULLETIN
DE
L'ACADÉMIE POLONAISE DES SCIENCES

SÉRIE DES SCIENCES
MATHÉMATIQUES, ASTRONOMIQUES ET PHYSIQUES

VOLUME VIII
NUMERO 2

VARSOVIE 1960

PRINTED IN POLAND

PAŃSTWOWE WYDAWNICTWO NAUKOWE, W-WA. MIODOWA 10

Nakład 1013+152 egz.	*Rękopis dostarczono 5.I.1960*
Ark. wyd. 3 ark. druk. 3,125+0,25	*Podpisano do druku 10.III.1960*
Papier druk. sat. 80 g kl. III	*Druk ukończono w marcu 1960*
Format B5, 70×100 cm	*Zam. prod. 3/60 Cena zł 20.—*

WARSZAWSKA DRUKARNIA NAUKOWA, W-WA, ŚNIADECKICH 8

BULLETIN DE L'ACADÉMIE
POLONAISE DES SCIENCES
Série des sci. math., astr.
et phys.—Vol. VIII, No. 2, 1960

MATHEMATICS

Banach Spaces Non-Isomorphic to Their Cartesian Squares. I

by

C. BESSAGA and A. PEŁCZYŃSKI

Presented by S. MAZUR on November 6, 1959

Banach *) raised the following question: if X is an infinite-dimensional B-space, is it necessarily isomorphic **) to its Cartesian square $X^2 = X \times X$.

In this note the question of Banach is answered in the negative by suitable examples. There exists namely a separable Banach space of infinite dimensions which is not isomorphic to its Cartesian square. Moreover, an example may be chosen such that $\dim_l X^2 > \dim_l X$ ***) as well as such that $\dim_l X^2 = \dim_l X$.

These examples can be constructed by the use of an arbitrary non-reflexive B-space X such that the deficiency of the canonical image of X in X^{**} is a finite positive number, for instance by the use of the space introduced by James in [8].

1. General considerations. In the sequel, X will denote a B-space, X^* and X^{**} — its first and second conjugate spaces, $x, y\ z, \ldots\ x^*, y^*, z^*, \ldots\ x^{**}, y^{**}, z^{**}, \ldots$ will denote elements of X, X^*, X^{**}, respectively.

$\mathcal{E}^n$ will denote the n-dimensional Euclidean space, C — the space of all continuous functions on $\langle 0,1\rangle$. $X \times Y$ will denote the Cartesian product of X and Y (with the norm $\|(x, y)\| = \|x\| + \|y\|$ and with usual linear operations) and $X^2 = X \times X$. Next, $X \sim Y$ will mean that X and Y are isomorphic and $X = Y$ will mean that X and Y are isometrically isomorphic.

By $\hat{X}$ we shall denote the canonical image of X in X^{**}.

By $\delta(X)$ we shall denote the deficiency of $\hat{X}$ in X^{**}, *i.e.* the dimension of the quotient space $X^{**}/\hat{X}$.

---

*) See [1], p. 245. This problem is also quoted by Kaplansky ([9], p. 5).

**) By isomorphism we understand any one-to-one linear bicontinuous mapping. The terminology used in this note is found in the monographs of Banach [1] and of Dunford and Schwartz [5].

***) $\dim_l X \leqslant \dim_l Y$ means that X is isomorphic to a subspace of Y; $\dim_l X = \dim_l Y$ means that $\dim_l X \leqslant \dim_l Y$ and $\dim_l Y \leqslant \dim_l X$ and $\dim_l X < \dim_l Y$ means that $\dim_l X \leqslant \dim_l Y$ without $\dim_l X = \dim_l Y$ (see [1], p. 193).

LEMMA 1. *If* $\delta(X)=n$ *and* $\delta(Y)=m$, *then* $\delta(X\times Y)=n+m$.

LEMMA 2. *Let* X *be a separable B-space with* $\delta(X)=n$, *and let* Y *be a subspace of* X. *Then* $\delta(Y)\leqslant n$.

Proof. We shall prove that, for any system of functionals $y_1^{**},\ldots,y_{n+1}^{**}$ belonging to Y^{**}, there exist numbers $\lambda_1,\ldots,\lambda_{n+1}$ such that $\sum|\lambda_i|\neq 0$ and $\sum\lambda_i y_i^{**}\in\hat{Y}$.

Let $y_i^{**}\in Y^{**}$, $i=1,2,\ldots,n+1$. Since X is separable, $X^{**}\sim X\times\mathcal{E}^n$ and X^* are separable, too ([1], p. 189).

Thus, Y^* is separable and, by a theorem of Gantmacher and Šmulyan [6], there exist sequences $(y_m^{(i)})$ in Y $(i=1,\ldots,n+1)$ such that

$$\lim_m y^*(y_m^i)=y_i^{**}(y^*) \quad \text{for all} \quad y^*\in Y \quad \text{and} \quad i=1,\ldots,n+1.$$

Hence, $(y_m^{(i)})$ are weak Cauchy sequences in Y and, consequently, they are weak Cauchy sequences in X. It follows that the limits

$$x_i^{**}(x^*)=\lim_m x^*(y_m^{(i)})$$

exist and $x_i^{**}\in X^{**}$ for $i=1,2,\ldots,n+1$. Since the deficiency of $\hat{X}$ in X^{**} is equal to n, we have $\sum_{i=1}^{n+1}\lambda_i x_i^{**}\in\hat{X}$ with $\sum_{i=1}^{n+1}|\lambda_i|\neq 0$, λ_i being constant. Thus, the sequence $\sum_{i=1}^{n+1}\lambda_i y_m^{(i)}$ converges weakly to the element z of X corresponding to $\sum_{i=1}^{n+1}\lambda_i x_i^{**}$. Since Y is weakly closed in X ([1], p. 134), z belongs to Y and

$$\sum_{i=1}^{n+1}\lambda_i y_i^{**}(y^*)=\lim_m y^*\left(\sum_{i=1}^{n+1}\lambda_i y_m^{(i)}\right)=y^*(z) \quad \text{for all} \quad y^*\in Y^{**},$$

which means that $\sum_{i=1}^{n+1}\lambda_i y_i^{**}\in\hat{Y}$.

THEOREM 1. *Let* X *be a separable B-space and let* $\delta(X)=n$ $(1\leqslant n<\infty)$. *Then* $\dim_l X<\dim_l X^2$.

Proof. Inequality $\dim_l X<\dim_l X^2$ is trivial; inequality $\dim_l X\geqslant\dim_l X^2$ is impossible in virtue of Lemmas 1 and 2, the quantity $\delta(X)$ being invariant with respect to isomorphisms.

THEOREM 2. *Let* X *be a weakly complete B-space and let* Y *be a separable B-space with* $\delta(Y)=n$ $(1\leqslant n<\infty)$. *Then the space* $X\times Y$ *and* $(X\times Y)^2$ *are not isomorphic.*

Proof. Let us consider the following property of the space Z:

(*) Let $z_k^{(i)}$ $(i=1,2,\ldots,n+1)$ be weak Cauchy sequences in Z. Then there exist numbers $\lambda_1,\ldots,\lambda_{n+1}$ such that $\sum_{i=1}^{n+1}|\lambda_i|\neq 0$ and that the sequence $\sum_{i=1}^{n+1}\lambda_i z_k^{(i)}$ converges weakly (as $k\to\infty$) to an element of Z.

1° The space $Z_1 = X \times Y$ has the property (*).

Proof. Let $z_k^{(i)} = (x_k^{(i)}, y_k^{(i)})$ $(i = 1, 2, ..., n+1)$ be weak Cauchy sequences in Z. Then $(x_k^{(i)})$ and $(y_k^{(i)})$ are weak Cauchy sequences in X and Y, respectively. X being weakly complete, $(x_k^{(i)})$ are weakly convergent to some elements $x_0^{(i)}$ $(i = 1, 2, ..., n+1)$. Next, the canonical images $\hat{y}_k^{(i)}$ of $y_k^{(i)}$ (in Y^{**}) converge $*$-weakly to some functionals y_i^{**} $(i = 1, 2, ..., n+1)$. According to the fact that $\delta(Y) = n$, there exist numbers $\lambda_1, \lambda_2, ... \lambda_{n+1}$ such that $\sum_{i=1}^{n+1} |\lambda_i| \neq 0$ and $\sum_{i=1}^{n+1} \lambda_i y_i^{**} = \hat{y} \in \hat{Y}$. Obviously, the sequence $\sum_{i=1}^{n+1} \lambda_i y_k^{(i)}$ converges weakly to y.

2° The space $Z_2 = (X \times Y)^2$ does not have the property (*).

Proof. Since $\delta(Y^2) = 2n \geqslant n+1$, there exist functionals $y_1^{**}, y_2^{**}, ..., y_{n+1}^{**}$ belonging to $(Y^2)^{**}$ such that, for every $0 \neq y_0^{**} \in \hat{Y}^2$, the functionals $y_0^{**}, ..., y_{n+1}^{**}$ are linearly independent. By the quoted theorem of Gantmacher and Šmulyan, there exist sequences $(y_k^{(i)})$ in Y^2 such that

$$y_i^{**}(y^*) = \lim_{k \to \infty} y^*(y_k^{(i)})$$

for all $y^* \in (Y^2)^*$ and for $i = 1, 2, ..., n+1$. Obviously, the sequences $z_k^{(1)} = (0, y_k^{(1)}), ..., z_k^{(n+1)} = (0, y_k^{(n+1)})$ do not satisfy the condition described in (*).

2. The spaces J and $C \times J$. The space J of James [8] consists of all real sequences $x = \{t_n\}$ such that

$$\lim_{n=\infty} t_n = 0, \tag{1}$$

$$\|x\| = \sup \left[(t_{p_1} - t_{p_2})^2 + \cdots + (t_{p_{n-1}} - t_{p_n})^2 + (t_{p_n} - t_{p_1})^2\right]^{1/2} < +\infty, \tag{2}$$

where sup denotes the l.u.b. extended on all finite increasing systems of indices $p_1 < p_2 < ... < p_n$ $(n = 1, 2, ...)$. The norm of J is given by (2).

James proved that J is a separable non-reflexive B-space, that J and J^{**} are isometric and that $\delta(J) = 1$. Thus, by Theorem 1, the spaces J and J^2 are not isomorphic and $\dim_l J < \dim_l J^2$.

Now, it is easily seen that the deficiency of the canonical image of $\hat{J}^*$ in J^{***} is also equal to 1; since the space C^* (conjugate to C) is weakly complete (Banach and Mazur [2], p.104), by Theorem 2 the space $C^* \times J^*$ is not isomorphic with its square. Since $(C \times J)^* = C^* \times J^*$, it follows that $C \times J$ and $(C \times J)^2$ are not isomorphic. On the other hand, these spaces have the same linear dimension, since, by the Banach-Mazur theorem ([1], p. 185), $\dim_l (C \times J) \leqslant \dim_l (C \times J)^2 \leqslant \dim_l C \leqslant \dim_l (C \times J)$.

3. Remarks. 1°. We shall say that a real B-space X has a complex structure if there exists an isomorphism (homogeneous with respect to real scalars) transforming X on a complex B-space Y. It is easily seen that if X is isomorphic to the square Z^2 of a real B-space Z, then X has

a complex structure. Indeed, the multiplication by complex numbers may be introduced in Z^2 as follows:

$$(\alpha+\beta i)\ (z', z'') = (\alpha z' - \beta z'',\ \alpha z'' + \beta z').$$

Now, Dieudonné [4] proved that the space J has not a complex structure, whence J is not isomorphic to the square of any B-space Z.

Lemmas 1 and 2 was proved by Civin and Yood [7].

2° PROBLEM. *Does there exist an infinitely dimensional reflexive B-space (having an unconditional basis) which is not isomorphic to its square? In particular, let X denote the space of the real sequences (t_n) for which*

$$\|(t_n)\| = \left[\sum_{n=1}^{\infty} \sum_{k=n\,(n+1)/2}^{(n+1)\,(n+2)/2} |t_k|^{2n}\right]^{1/2} < \infty,$$

provided with the norm $\|\cdot\|$. Is X isomorphic to its square?

3° Every infinitely dimensional *L-space* *) is isomorphic with its Cartesian square. This follows by Maharam's decomposition theorem in a way similar to that used by Day [3].

4° One may easily show that if X is a B-space and if $X \sim X^2$ and $(X \times \mathcal{C}^1) \sim (X \times \mathcal{C}^1)^2$, then $X \sim X \times \mathcal{C}^1$. Indeed, then

$$X \times \mathcal{C}^1 \sim (X \times \mathcal{C}^1)^2 \sim X^2 \times \mathcal{C}^2 \sim X \times \mathcal{C}^2,$$

whence $X \sim X \times \mathcal{C}^1$ **).

However, Banach's problem, whether every infinite-dimensional B-space X is isomorphic to its maximal subspace (equivalently, whether $X \sim X \times \mathcal{C}^1$), is still open, even if $X \sim X^2$.

INSTITUTE OF MATHEMATICS, POLISH ACADEMY OF SCIENCES
(INSTYTUT MATEMATYCZNY, PAN)

REFERENCES

[1] S. Banach, *Théorie des opérations linéaires*, Warsaw, 1932.

[2] S. Banach und S. Mazur, *Zur Theorie der linearen Dimensionen*, Studia Math., **4** (1933), 100—112.

[3] M. M. Day, *Every L-space is isomorphic to a strictly convex space*, Proc. Amer. Math. Soc., **8** (1957), 415—417.

[4] J. Dieudonné, *Complex structures on real Banach spaces*, Proc. Amer. Math. Soc., **3** (1952), 162—164.

[5] N. Dunford and J. Schwartz, *Linear Operators* I, London, 1958.

[6] W. I. Gantmacher et V. Šmulyan, *Sur les espaces linéaires dont la sphère unitaire est faiblement compacte*, Dokl. AN SSSR, **17** (1937), 91—94.

[7] P. Civin and B. Yood, *Quasi-reflexive Spaces*, Proc. Amer. Math. Soc. **8** (1957), 906—911.

[8] R. C. James, *A non-reflexive Banach space isometric with its second conjugate space*, Proc. Nat. Acad. Sci., **37** (1951), 174—177.

[9] I. Kaplansky, *Functional analysis, some aspects of analysis and probability*, New York, 1958.

*) A Banach lattice is called an L-space, if $x \geqslant 0$, $y \geqslant 0$ imply $\|x\|+\|y\| = \|x+y\|$.

**) All maximal hyperplanes of a B-space X are isomorphic. Indeed, let $x_1^* \in X^*$, $x_2^* \in X^*$ and $x_1^* \neq 0$, $x_2^* \neq 0$, and let $H_i = \{x : x_i^*(x) = 0\}$ for $i = 1, 2$. Then

$$H_1 \sim (H_1 \cap H_2) \times \mathcal{C}^1 \sim H_2.$$

BULLETIN DE L'ACADÉMIE
POLONAISE DES SCIENCES
Série des sci. math., astr.
et phys.—Vol. VIII, No. 2, 1960

MATHEMATICS

Banach Spaces Non-Isomorphic to Their Cartesian Squares. II

by

Z. SEMADENI

Presented by W. ORLICZ on December 1, 1959

Recently, Bessaga and Pełczyński [4] have given the negative solution of a problem of Banach ([2], p. 245); namely, they have established the existence of an infinite-dimensional B-space X which is not isomorphic to its Cartesian sqare $X^2 = X \times X$. In this note another example of such a B-space is given.

Let Γ_{ω_1} be the set of all ordinals lesser or equal to the first uncountable ordinal ω_1; Γ_{ω_1} is a non-metrisable compact Hausdorff space in the order topology. We shall prove that the space $C(\Gamma_{\omega_1})$ of real-valued continuous functions on Γ_{ω_1} (with the usual norm) is not isomorphic to its Cartesian square.

Notation. X will denote a Banach space, X^* and X^{**} — its first and second conjugate spaces, x, y, ..., x^*, y^*, ..., x^{**}, y^{**}, ... — elements of X, X^*, X^{**}, respectively. $X \sim Y$ will mean that X is isomorphic to Y.

$\omega = \omega_0, \omega_1, \omega_\tau, \omega_\mathfrak{c}$ will denote the first ordinals of power $\aleph_0, \aleph_1, \aleph_\tau, \mathfrak{c} = 2^{\aleph_0}$, respectively. α being an ordinal, Γ_α will denote the set $\{\beta : \beta \leqslant \alpha\}$ with the order topology. $\delta_{t't}$ will be Kronecker's delta: $\delta_{t't'} = 1$ and $\delta_{t't} = 0$ for $t' \neq t$.

$c_0(\aleph_\tau)$ will denote the space of all real-valued functions on an isolate set of power $\aleph_\tau$, vanishing at infinity, with usual norm.

In this paper Ω will denote a dispersed (= clairsemé) compact Hausdorff space and $C(\Omega)$ will denote the space of all real-valued continuous functions on Ω, with $\|x\| = \sup\{|x(t)| : t \in \Omega\}$. We shall write shortly C_α instead of $C(\Gamma_\alpha)$. By a theorem of Rudin ([8], see also [7]), every linear functional on $C(\Omega)$ is of the form

$$x^*(x) = \sum_{t \in \Omega} a(t)\, x(t) \qquad \text{with} \qquad \|x^*\| = \sum_{t \in \Omega} |a(t)|,$$

where, for any x^*, the set $\{t: a(t) \neq 0\}$ is countable. Thus, the space X^* conjugate to $X = C(\Omega)$ may be identified with the space $l^1(\Omega)$ of all

functions $a(t)$ on Ω for which $\sum |a(t)| < \infty$. In turn, every linear functional on $l^1(\Omega)$ is of the form

$$x^{**}(x^*) = \sum a(t)\, u(t) \quad \text{with} \quad \|x^{**}\| = \sup\{|u(t)| : t \in \Omega\},$$

whence the space X^{**} is equivalent to the space $m(\Omega)$ of all bounded real-valued functions on Ω.

A functional x^{**}, corresponding to a bounded function $u(t)$ on Ω, belongs to the *canonical image* $\hat{X}$ of X in X^{**} if and only if $u(t)$ is continuous on Ω.

1. The set X_s. Given a B-space X, X_s will denote the set of all linear functionals on X^* sequentially continuous with respect to the *-weak topology $\sigma(X^*, X)$. Thus, $x^{**} \in X_s$ if and only if the condition $x_n^*(x) \to 0$ *for all* $x \in X$ implies $x^{**}(x_n^*) \to 0$. Obviously, $\hat{X} \subset X_s \subset X^{**}$. In order that $x^{**} \in X_s$, it is necessary and sufficient that the hyperplane $H(x^{**}) = \{x^* : x^{**}(x^*) = 0\}$ be sequentially closed in the topology $\sigma(X^*, X)$ *).

LEMMA 1. *The set X_s is strongly closed in X^{**}.*

We omit the easy proof. By $m_s(\Omega)$ we shall denote the set of all functions $u \in m(\Omega)$ corresponding to functionals in X_s, where $X = C(\Omega)$.

LEMMA 2. *Let γ be a limit ordinal non-cofinal with ω, and let $\alpha \geqslant \gamma$. Then the function $u_\gamma(\beta) = \delta_{\gamma\beta}$ belongs to $m_s(\Gamma_\alpha)$.*

Proof. We shall prove that the hyperplane $H(x^{**})$ is sequentially *-weakly closed. Obviously, $H(x^{**})$ consists of all functionals x^* corresponding to sequences $\{a_\beta\}$ such that $x^{**}(x^*) = \sum_{\beta \leqslant \alpha} a_\beta u_\gamma(\beta) = a_\gamma = 0$.

Let $y_n^* \in H(x^{**})$ and let $y_n^*(x) = \sum b_\beta^{(n)} x_\beta \to y_0^*(x) = \sum b_\beta^{(0)} x_\beta$ for all $x \in C_\alpha$, $n \to \infty$. Since the set $\{\beta : \sup_{n=0,1,\ldots} |b_\beta^{(n)}| \neq 0\}$ is countable, there exists an ordinal $\gamma_1 < \gamma$ such that $b_\beta^{(n)} = 0$ for $\gamma_1 < \beta < \gamma$ and for $n = 0, 1, 2, \ldots$ Denote $z_0 = \{z_\beta^{(0)}\}$, where $z_\beta^{(0)} = 1$ for $\gamma_1 < \beta \leqslant \gamma$ and $z_\beta^{(0)} = 0$ elsewhere. Then $z_0 \in C(\Gamma_\alpha)$ and $y_n^*(z_0) = 0$ for $n = 1, 2, \ldots$. Hence, $y_0^*(z_0) = b_\gamma^{(0)} = 0$ which means that $y_0^* \in H(x^{**})$. Thus, we have proved that $u_\gamma(\cdot) \in m_s(\Gamma_\alpha)$.

THEOREM 1. *Let $\alpha = \omega_1 \cdot \gamma$ with $\gamma \geqslant 1$, and let $X = C(\Gamma_\alpha)$. Then the functional x^{**}, corresponding to a bounded sequence $\{u_\beta\}$, belongs to X_s if and only if the sequence $\{u_\beta\}$ is continuous at every limit ordinal β_0 cofinal with ω.*

Proof. **Necessity.** Let $x^{**} \in X_s$, let x^* be a functional such that $x^{**}(x^*) = \sum u_\beta a_\beta = 1$, and suppose, if possible, that $\{\beta_n\}$ is a sequence of ordinals such that $\beta_1 < \beta_2 < \ldots$, $\beta_n \to \beta_0$ as $n \to \infty$ and $\lim u_{\beta_n} = \mu \neq u_{\beta_0}$. We shall prove that $\{u_\beta\} \notin m_s(\Gamma_\alpha)$. Let us denote

$$y_m^*(x) = \sum_{\beta \leqslant \alpha} b_\beta^{(m)} x_\beta \quad \text{and} \quad z_m^* = \frac{1}{u_{\beta_0} - \mu}\, y_m^* + \frac{u_{\beta_0} - u_{\beta_m} - 1}{u_{\beta_0} - \mu}\, x^*,$$

*) Equivalence between sequential continuity of an additive functional and sequential closedness of its nul-hyperplane is valid for any $\mathcal{L}$-convergence in the sense of Fréchet (see [1], p. 129).

where

$$b_\beta^{(m)} = \begin{cases} a_\beta & \text{if} \quad \beta \neq \beta_n \quad \text{for} \quad n = 0, 1, 2, \ldots, \\ \delta_{nm} + a_{\beta_n} & \text{if} \quad \beta = \beta_n, \ n = 1, 2, \ldots, \\ a_{\beta_0} - 1 & \text{if} \quad \beta = \beta_0. \end{cases}$$

Then $y_m^*(x) = \sum a_\beta x_\beta + x_{\beta_m} - x_{\beta_0} = x^*(x) + x_{\beta_m} - x_{\beta_0} \to x^*(x)$ as $m \to \infty$, whence $z_m^*(x) \to x^*(x)$. Moreover, $x^{**}(y_m^*) = \sum u_\beta b_\beta^{(m)} = \sum u_\beta a_\beta + u_{\beta_m} - u_{\beta_0} = 1 + u_{\beta_m} - u_{\beta_0}$, whence $x^{**}(z_m^*) = 0$ for $m = 1, 2, \ldots$. Thus, $z_m^* \in H(x^{**})$ and $x^* \in H(x^{**})$; accordingly, $x^{**} \notin X_s$.

Sufficiency. Let $\{u_\beta\}$ be continuous at every point G_δ of Γ_α. We shall prove that $x^{**} \in X_s$. Obviously, $\lim_{\beta \to \beta_0} u_\beta$ exists for every $\beta_0 \in \Gamma_\alpha$. Next, since every infinite set of ordinals contains a sequence convergent to an ordinal cofinal with ω, for any $\varepsilon > 0$ the set

$$A_\varepsilon = \{\beta : \beta \in \Gamma_\alpha, \ |\lim_{\varphi \to \beta} u_\varphi - u_\beta| \geq \varepsilon\}$$

must be finite. Thus, we may arrange the points of discontinuity of $\{u_\beta\}$ into a sequence $\beta_1, \beta_2, \ldots$, and, denoting $\lambda_n = \lim_{\varphi \to \beta_n} u_\varphi - u_{\beta_n}$, we conclude that $\lambda_n \to 0$ as $n \to \infty$. It is easily seen that the sequence

$$w_\beta^{(n)} = \lambda_1 \delta_{\beta_1 \beta} + \cdots + \lambda_n \delta_{\beta_n \beta} \qquad (n = 1, 2, \ldots)$$

is uniformly convergent on Γ_α and that the sequence

$$v_\beta = u_\beta + \lim_n w_\beta^{(n)}$$

is continuous on Γ_α. Thus, by Lemmas 1 and 2, $\{u_\beta\} \in m_s(\Gamma_\alpha)$.

2. Main theorem. Obviously, the set X_s is defined by isomorphic invariants; accordingly, its properties may be used for isomorphic classification of certain B-spaces.

THEOREM 2. *If $X = C_{\omega_1 \cdot n}$ and if $1 \leq n < \omega$, then the quotient space $X_s/\hat{X}$ is isomorphic to C_n. If $X = C_{\omega_\tau \cdot \gamma}$ with $\omega_\tau \leq \gamma < \omega_{\tau+1}$ and $\tau \geq 0$, then $X_s/\hat{X}$ is isomorphic to $c_0(\aleph_\tau)$. Thus, the spaces*

$$C_{\omega_1}, C_{\omega_1 \cdot 2}, \ldots C_{\omega_1 \cdot \omega}, C_{\omega_2 \cdot \omega}$$

are mutually non-isomorphic.

The space $C_{\omega_1 \cdot 2n}$ being equivalent to $C_{\omega_1 \cdot n} \times C_{\omega_1 \cdot n}$ (provided with the norm $\|(x, y)\| = \max(\|x\|, \|y\|)$), the space C_{ω_1}, as well as each space $C_{\omega_1 \cdot n}$, is not isomorphic to its Cartesian square.

The first part of Theorem 2 is a consequence of Theorem 1. The second is an easy consequence of a localization theorem (see [9]) and the detailed proof will be published elsewhere.

3. Remarks. 1° Theorem 2 gives the negative solution of a problem raised by Bessaga and Pełczyński in [3]. At the same time, it extends their classification of isomorphic types of the spaces C_α, established for $\alpha < \omega_1$. Since

$$\Gamma_{\omega_1 \cdot n + \varphi} \underset{\text{top}}{=} \Gamma_{\omega_1 \cdot n} \qquad \text{for} \qquad \varphi < \omega_1, \ n = 1, 2, \ldots,$$

Theorem 2 gives a complete classification of isomorphic types of the spaces

C_α for $\alpha \leqslant \omega_1 \cdot \omega$. Unfortunately, the presented method does not give full solution of the problem.

2° In both examples (J and $C \times J$) considered by Bessaga and Pełczyński [4], the conjugate spaces (J^* and $C^* \times J^*$) are also non-isomorphic to their Cartesian squares; however, the space $l^1(\Gamma_{\omega_1})$ and $l^1(\Gamma_{\omega_1 \cdot 2})$, conjugate to C_{ω_1} and $C_{\omega_1 \cdot 2}$, are equivalent.

3° Given a B-space X and a functional $x^{**} \in X^{**} \setminus \hat{X}$, the hyperplane $H = \{x^* : x^{**}(x^*) = 0\}$ is norming for X, i.e. the norm

$$\|x\|_H = \sup \{x^*(x) : x^* \in H, \|x^*\| = 1\}$$

is equivalent to the initial norm $\| \|$ in X. Hence, H is dense in X^* with respect to the bounded *-weak topology of X^* (Dixmier [5]). If, moreover, X is separable, then H is sequentially *-weakly dense in X^* (Banach [2], p. 124). Consequently, if X is separable, then $X_s = \hat{X}$.

Now, Grothendieck ([6], p. 168) proved that if K is a compact extremally disconnected Hausdorff space and if $X = C(K)$, then the *-weak convergence of a countable sequence of functionals $x_n^* \in X^*$ is equivalent to the weak convergence (with respect to X^{**}). Hence, in this case $X_s = X^{**}$.

4° A. Pełczyński has remarked that, in the case $X = C_{\omega_1 \cdot n}$, the set X_s is identical with the set X_w of all linear functionals on X^* satisfying the following condition:

(W) { for every sequence x_n^* in X^* there exists an element $x \in X$ such that $x^{**}(x_n^*) = x_n^*(x)$ for $n = 1, 2, \ldots$

This property is also an isomorphic invariant. Moreover, Pełczyński has proved *) that, for any subspace Y of C_{ω_1}, the deficiency of $\hat{Y}$ in Y_w is $\leqslant 1$ and, consequently,

$$\dim_l C_{\omega_1} < \dim_l C_{\omega_1 \cdot 2}.$$

DEPARTMENT OF MATHEMATICS, A. MICKIEWICZ UNIVERSITY, POZNAŃ
(KATEDRA MATEMATYKI, UNIWERSYTET A. MICKIEWICZA, POZNAŃ)

REFERENCES

[1] A. Alexiewicz and Z. Semadeni, *Linear functionals on two-norm spaces*, Studia Math., **17** (1958), 121—140.

[2] S. Banach, *Théorie des opérations linéaires*, Warszawa, 1932.

[3] C. Bessaga and A. Pełczyński, *Spaces of continuous functions* (IV), Studia Math., in press.

[4] C. Bessaga and A. Pełczyński, *Banach spaces non-isomorphic to its Cartesian squares*, I., this issue.

[5] J. Dixmier, *Sur un théorème de Banach*, Duke Math. J., **15** (1948), 1057—1071.

[6] A. Grothendieck, *Sur les applications linéaires faiblement compacts d'espace du type C(K)*, Canad. J. Math., **5** (1953), 129—173.

[7] A. Pełczyński and Z. Semadeni, *Spaces of continuous functions* (III), Studia Math., **18** (1959), 211—222.

[8] W. Rudin, *Continuous functions on compact spaces without perfect subsets*, Proc. Amer. Math. Soc., **8** (1957), 39—42.

[9] Z. Semadeni, *A localization theorem for multiplicative linear functionals*, Bull. Acad. Polon. Sci., Sér. sci., math., astr. et phys. 6 (1938), 289—292.

*) The proof will be published later.

BULLETIN DE L'ACADÉMIE
POLONAISE DES SCIENCES
Série des sci. math., astr.
et phys.—Vol. VIII, No. 2, 1960

The Molar Refraction of π-Electrons of Some Conjugated Molecules

by

H. SUGIER and W. WOŹNICKI

Presented by A. JABŁOŃSKI on December 3, 1959

The molar refraction is a property defined by

$$R = \frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d}, \tag{1}$$

where n is the refraction index, M — molecular weight, d — density.

As a rule, molar refraction is an additive quantity and is computed as the sum of suitable atomic and bond contributions. There are, however, many compounds which do not belong to this additive scheme, in the first place those with conjugated double bonds. Their molar refractions are greater than the values calculated from the table of atomic refractions. This difference is an optical exaltation.

Since the molar refraction is directly connected with the mean polarizability α of the electrons in molecules

$$R = \frac{4}{3} \pi \alpha N \qquad (N \text{ — Avogadro number}), \tag{2}$$

it seems that the anomalously high refraction of such molecules may be ascribed to π-electrons, which in the systems of the conjugated bonds are delocalized in a high degree and sensitive to the influence of the electric field. Therefore, it is reasonable to divide the molar refraction into the refraction of the core R_c, which should remain in the additive scheme, and the refraction of π-electrons R_π, which consists of increments of double bonds and optical exaltation

$$R = R_c + R_\pi. \tag{3}$$

The π-electrons refraction may be calculated from (2) if appropriate wave functions are known, namely

$$\alpha_\pi = \frac{e^2}{4\pi^2 m} \sum_{i,k} \frac{f_{ik}}{\nu_{ik}^2 - \nu^2}, \tag{4}$$

where f_{ik} is the oscillator strength of electronic transition

$$f_{ik} = 2 \cdot \frac{8\pi^2 m}{3h} \nu_{ik} \vec{r}_{ik}^{\,2}, \tag{5}$$

$\vec{r}_{ik}$ being the corresponding matrix element

$$\vec{r}_{ik} = \int \Psi_i^* \vec{r} \Psi_k \, d\tau. \tag{6}$$

In (4) ν_{ik} is the frequency of corresponding transition, ν — the frequency of light for which the molar refraction is determined.

The wave functions of π-electrons may be obtained on the basis of the FEMO model considering the potential along the core of the molecule as a constant. On account of non equivalency of the bonds in the polyene chain it seems necessary, however, to take into consideration the pe-

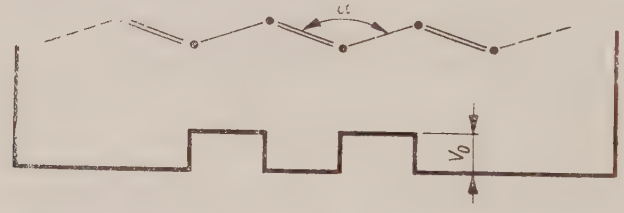


Fig. 1. The potential along the chain in a hexatriene molecule ($\alpha = 124°$)

riodical potential [1]. For the sake of simplicity the potential of the rectangular type has been assumed. The height of the potential barrier V_0 has been adapted on the ground of experimental value of the longest wave transition in butadiene: $V_0 = 6.6\ eV$.

The fitness of such a potential is confirmed by the results presented in Table I.

TABLE I

The spectra of polyenes (the first absorption band)

	$V_0 = 0$		$V_0 = 6.6\ eV$		experimental	
	ν[cm^{-1}]	f	ν[cm^{-1}]	f	ν[cm^{-1}]	f
butadiene	30800	0.97	48000	0.79	48000	0.53
hexatriene	22000	1.42	40700	1.12	40500	0.62

In the first two columns the wave numbers and oscillator strength calculated on the basis of free electron model ($V_0 = 0$) are given *). The next two columns contain analogous values for $V_0 = 6.6$ eV. In the last two experimental data are presented [2], [3].

*) For $V_0 = 0$ the oscillator strength in the polyenes series has been calculated by Bayliss [2]. This calculations seems, however, to be doubtful. Namely, the length of the free π-electrons path adapted on the basis of the experimental value of ν, is in many cases lesser, than the real chain length. Besides, the authors did not consider the geometrical structure of molecule and omitted in the expression (5) factor 2.

Table II contains the calculated molar refractions of certain simple hydrocarbons with conjugated bonds. Beside the values of the refractions for a given light wave (D) or (α), static refractions ($\lambda \to \infty$) are presented. The latter are better qualified for the estimation of the exaltation effect, because in the vicinity of the absorption band the dispersion formula overestimates the refraction values.

From Table II it is evident that, by the consideration of the interaction potential of π-electrons with the core of the molecule our results have been markedly improved. This proves that the introduction of a potential reducing the mobility of π-electrons in conjugated molecules with non equivalent bonds is justified. The free electron model ($V_0 = 0$) ascribes to π-electrons a too high sensitivity to the influence of the electric field,

TABLE II

The molar refractions of π-electrons [in cm^3]

	calculated $V_0 = 0$		calculated $V_0 = 6{,}66\ eV$		from spectrum data		
	$R_{\pi st}$	$R_{\pi\lambda}$	$R_{\pi st}$	$R_{\pi\lambda}$	$R_{\pi st}$	$R_{\pi\lambda}$	$R_{\pi exp}$
butadiene	18.35	26.33 (D)	6.16	7.04 (D)	4.13	4.71 (D)	4.89 (D)
hexatriene	52.82	101.6 (α)	12.20	14.20 (α)	6.80	7.91 (α)	7.39 (α)
hexadiene	18.35	26.33 (D)	6.16	7.04 (D)	—		5.13 (D)
1,3-cyclo-hexadiene	5.99	8.60 (D)	1.80	2.04 (D)	—	—	3.44 (D)

producing molar refraction values much higher than those observed. Table II contains also the refraction values calculated from (4) and (5) on the ground of experimental spectral data. Their good agreement with the experimental values confirms the possibility of the estimation of the exaltation effect from the dispersion formula. It follows also that, with correct wave functions of π-electrons, results in complete agreement with experiment could be obtained.

DEPARTMENT OF PHYSICAL CHEMISTRY, TECHNICAL UNIVERSITY, ŁÓDŹ
(KATEDRA CHEMII FIZYCZNEJ, POLITECHNIKA, ŁÓDŹ)
DEPARTMENT OF THEORETICAL PHYSICS, NICHOLAS COPERNICUS UNIVERSITY, TORUŃ
(KATEDRA FIZYKI TEORETYCZNEJ, UNIWERSYTET MIKOŁAJA KOPERNIKA, TORUŃ)

REFERENCES

[1] H. Kuhn, J. Chem. Phys., **17** (1949), 1198.
[2] N. S. Bayliss, J. Chem. Phys., **16** (1948), 287.
[3] J. R. Platt, H. B. Klevens, Rev. Mod. Phys., **16** (1944), 182.

BULLETIN DE L'ACADÉMIE
POLONAISE DES SCIENCES
Série des sci. math., astr.
et phys.—Vol. VIII, No. 2, 1960

Tight Binding Method for White Tin

by

M. MIĄSEK

Presented by L. INFELD on December 5, 1959

One of the methods of calculating the energy bands in solids is the tight binding or the LCAO method initiated by Bloch. This method and its applications were worked out in detail by Slater and Koster [1].

The tables of the matrix components of energy were prepared by Slater and Koster for the following lattices: (*a*) simple cubic structure, (*b*) face-centered cubic structure, (*c*) body-centered cubic structure, (*d*) diamond structure. In the cases (*a*), (*b*) and (*c*) the calculations were performed for *s*, *p* and *d* states, in the case (*d*) only for *s* and *p* states. For the hexagonal close-packed structure, for the states *s*, *p* and *d*, the tables were calculated by the author of this paper [2], [3].

Here, we intend to investigate the matrix components of energy for the white tin structure.

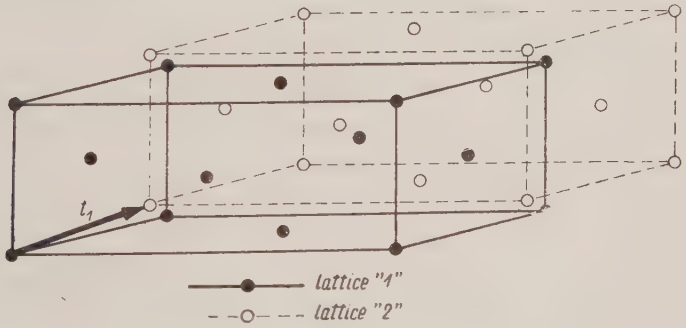


Fig. 1

The lattice of white tin can be described as composed of two inter-penetrating tetragonal face-centered lattices. The translation vectors for these lattices are $\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3$ where $|\boldsymbol{A}_1| = |\boldsymbol{A}_2| = a$, $|\boldsymbol{A}_3| = c$. For white tin we have $a = 8.25$ Å, $c = 3.17$ Å. The rectangular co-ordinate axes are taken along the $\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3$ vectors. Locating the origin of the co-ordinate system at the position of an atom we shall have a lattice "1" containing an atom at the position (0, 0, 0) and lattice "2" with the atom at the position $(\frac{1}{4}a, \frac{1}{4}a, \frac{1}{4}c)$. The lattice of white tin is shown in Fig. 1.

The lattice of white tin differs from the diamond lattice only by the distance between atoms in the direction z, namely for the white tin lattice $|\boldsymbol{A}_3| = c \neq a$ and for the diamond lattice $|\boldsymbol{A}_3| = a$.

Alternatively we can describe the white tin lattice as composed of two inter-penetrating tetragonal body-centered lattices.

In further calculations we shall use rectangular co-ordinate axes along the vectors $\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3$. These vectors are in principle not translation vectors of Bravais lattice as the cell built on these vectors is not the smallest elementary cell. It contains 8 nonequivalent atoms — 4 for the face-centered lattice "1" and 4 for lattice "2".

The translation vectors of the Bravais lattice are $\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3$.

$$\boldsymbol{a}_1 = \frac{1}{2}(-\boldsymbol{A}_1 + \boldsymbol{A}_3), \qquad \boldsymbol{a}_2 = \frac{1}{2}(\boldsymbol{A}_1 + \boldsymbol{A}_3), \qquad \boldsymbol{a}_3 = \frac{1}{2}(\boldsymbol{A}_2 - \boldsymbol{A}_3).$$

The components of these vectors in the actually used co-ordinate system are: $\boldsymbol{a}_1 = (-\frac{1}{2}a, 0, \frac{1}{2}c)$, $\boldsymbol{a}_2 = (\frac{1}{2}a, 0, \frac{1}{2}c)$, $\boldsymbol{a}_3 = (0, \frac{1}{2}a, -\frac{1}{2}c)$. The unit cell is built on these vectors. It contains two nonequivalent atoms in positions $(0, 0, 0)$ and $(\frac{1}{4}a, \frac{1}{4}a, \frac{1}{4}c)$.

The atomic volume is

$$V_a = \frac{1}{8}[\boldsymbol{A}_1 \cdot (\boldsymbol{A}_2 \times \boldsymbol{A}_3)] = \frac{1}{2}[\boldsymbol{a}_1 \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3)] = \frac{a^2 c}{8} = 27\ \text{Å}_3.$$

The matrix components of energy for white tin are $H_{mn}^{\omega\omega'}$. Here m, n denote four atomic states — s and three states p, because the electrons responsible for the physical properties of white tin are in states $5s$ and $5p$. ω and ω' correspond to two nonequivalent atoms in the unit cell. So we have $8 \times 8 = 64$ matrix components.

The general formula for the matrix components of energy is given by

$$H_{mn}^{\omega\omega'} = (m/n)_{\omega\omega'} = \exp[i\boldsymbol{k}(\boldsymbol{t}_\omega - \boldsymbol{t}_{\omega'})] \sum_l \exp(i\boldsymbol{k}\boldsymbol{r}_l) \times$$

$$\times \int \varphi_m^*(\boldsymbol{r} - \boldsymbol{t}_{\omega'})\, H\, \varphi_n(\boldsymbol{r} - \boldsymbol{r}_l - \boldsymbol{t}_\omega)\, d\boldsymbol{r},$$

where m, n denote atomic states, φ_n — atomic functions, $\boldsymbol{r}_l$ — translation vectors, $\boldsymbol{t}_\omega$ — basis vectors, H — the Hamiltonian of the whole system — is a Hermitian operator.

The energy integrals (E-integrals) have the form

$$E_{m,n}(\boldsymbol{t}_{\omega'}, \boldsymbol{r}_l + \boldsymbol{t}_\omega) = \int \varphi_m^*(\boldsymbol{r} - \boldsymbol{t}_{\omega'})\, H\, \varphi_n(\boldsymbol{r} - \boldsymbol{r}_l - \boldsymbol{t}_\omega)\, d\boldsymbol{r}.$$

Using the symmetry properties of the Hamiltonian and of the atomic functions we can perform the reduction of the matrix components. We obtain the relations between components of the type $(m/n)_{11}$ and $(m/n)_{22}$, and the $(m/n)_{12}$ and $(m/n)_{21}$. These relations for the states s and p are given in Table I of [2].

After carrying out this reduction we must further calculate only 20 components: 10 components of the type $(m/n)_{11}$ and 10 components of the type $(m/n)_{12}$.

We now make the nearest neighbours' approximation, namely we retain in sums $\sum_l$ in the matrix components only expressions with r_l's corresponding to the first and the second neighbours of an atom in the position (0, 0, 0). We have four first neighbours belonging to the lattice "2". Their position vectors, denoted by t_i, are: $\boldsymbol{t}_1 = (\frac{1}{4}a, \frac{1}{4}a, \frac{1}{4}c)$, $\boldsymbol{t}_2 = (-\frac{1}{4}a, \frac{1}{4}a, -\frac{1}{4}c)$, $\boldsymbol{t}_3 = (-\frac{1}{4}a, -\frac{1}{4}a, \frac{1}{4}c)$, $\boldsymbol{t}_4 = (\frac{1}{4}a, -\frac{1}{4}a, -\frac{1}{4}c)$. The distance from the first neighbour atom to the zero atom is equal to 3.02 Å. We have two second neighbours belonging to the lattice "1". Their position vectors are: $\boldsymbol{r}_1 = (0, 0, c)$, $\boldsymbol{r}_2 = (0, 0, -c)$. Their distance to zero atom is equal to 3.17 Å. The third neighbour atoms are at a distance 3.76 Å, so we omit them. In further calculations we shall write for vector components only $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ etc, without a and c.

We have now only three terms in sums for $(m/n)_{11}$ and four terms for $(m/n)_{12}$. Therefore, we have to calculate $3\times10+4\times10=70$ energy integrals. Not all of them are independent. Taking into account the symmetry transformations of the lattice, we can reduce the number of E-integrals. The Hamiltonian is invariant under a group of inhomogeneous orthogonal transformations carrying the lattice into itself. Under these transformations the atomic functions $\varphi_n(\boldsymbol{r}-\boldsymbol{r}_l)$ centered on atom $\boldsymbol{r}_l$ are centeerd on another atom, for instance on atom $\boldsymbol{r}_k$. Sometimes, the type of atomic function is also changed, for instance the p atomic function of the type x changes into p function of the type y.

For the reduction of E-integrals we have only to consider the subgroup of the homogeneous transformations. We have the following homogeneous orthogonal transformations of the lattice: 1) the identity transformation, 2) twofold rotation about vertical axis (z-axis), 3) the reflection in the plane $z=0$ and simultaneous rotation through the angle $\frac{1}{2}\pi$ about z-axis, 4) the reflection in the plane $z=0$ and simultaneous rotation through the angle $\frac{3}{2}\pi$ about z-axis, 5) reflection in the plane $y=x$, 6) reflection in the plane $y=-x$, 7) reflection in the plane $z=0$ and simultaneous reflection in the plane $x=0$, 8) reflection in the plane $z=0$ and simultaneous reflection in the plane $y=0$.

The relations between E-integrals are given in Table I. We remain with 15 independent E-integrals.

Now we can write the matrix components of energy in the nearest neighbours approximation. We use here the following abbreviations

$$\xi = \frac{1}{4}ak_x, \qquad \eta = \frac{1}{4}ak_y, \qquad \zeta = \frac{1}{4}ck_z.$$

We prepared Table II also in the two-centre approximation. The latter has been discussed in detail by Slater and Koster [1]. Using Table I of [1] we must know only the direction cosines of the vectors between

TABLE I

$$E_{s,x}(0,0,0) = E_{s,y}(0,0,0) = E_{s,z}(0,0,0) = E_{x,y}(0,0,0) = E_{x,z}(0,0,0) = E_{y,z}(0,0,0) = 0$$
$$E_{x,x}(0,0,0) = E_{y,y}(0,0,0)$$
$$E_{s,x}(0,0,1) = E_{s,x}(0,0,-1) = E_{s,y}(0,0,1) = E_{s,y}(0,0,-1) = 0$$
$$E_{x,z}(0,0,1) = E_{x,z}(0,0,-1) = E_{y,z}(0,0,1) = E_{y,z}(0,0,-1) = 0$$
$$E_{s,s}(0,0,1) = E_{s,s}(0,0,-1)$$
$$E_{s,z}(0,0,1) = -E_{s,z}(0,0,-1)$$
$$E_{x,x}(0,0,1) = E_{x,x}(0,0,-1) = E_{y,y}(0,0,1) = E_{y,y}(0,0,-1)$$
$$E_{z,z}(0,0,1) = E_{z,z}(0,0,-1)$$
$$E_{x,y}(0,0,1) = -E_{x,y}(0,0,-1)$$
$$E_{s,s}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}) = E_{s,s}(-\tfrac{1}{4},\tfrac{1}{4},-\tfrac{1}{4}) = E_{s,s}(-\tfrac{1}{4},-\tfrac{1}{4},\tfrac{1}{4}) = E_{s,s}(\tfrac{1}{4},-\tfrac{1}{4},-\tfrac{1}{4})$$
$$E_{s,x}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}) = -E_{s,x}(-\tfrac{1}{4},\tfrac{1}{4},-\tfrac{1}{4}) = -E_{s,x}(-\tfrac{1}{4},-\tfrac{1}{4},\tfrac{1}{4}) = E_{s,x}(\tfrac{1}{4},-\tfrac{1}{4},-\tfrac{1}{4})$$
$$E_{s,y}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}) = E_{s,y}(-\tfrac{1}{4},\tfrac{1}{4},-\tfrac{1}{4}) = -E_{s,y}(-\tfrac{1}{4},-\tfrac{1}{4},\tfrac{1}{4}) = -E_{s,y}(\tfrac{1}{4},-\tfrac{1}{4},-\tfrac{1}{4})$$
$$E_{s,x}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}) = E_{s,y}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4})$$
$$E_{s,z}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}) = -E_{s,z}(-\tfrac{1}{4},\tfrac{1}{4},-\tfrac{1}{4}) = E_{s,z}(-\tfrac{1}{4},-\tfrac{1}{4},\tfrac{1}{4}) = -E_{s,z}(\tfrac{1}{4},-\tfrac{1}{4},-\tfrac{1}{4})$$
$$E_{x,x}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}) = E_{x,x}(-\tfrac{1}{4},\tfrac{1}{4},-\tfrac{1}{4}) = E_{x,x}(-\tfrac{1}{4},-\tfrac{1}{4},\tfrac{1}{4}) = E_{x,x}(\tfrac{1}{4},-\tfrac{1}{4},-\tfrac{1}{4})$$
$$E_{y,y}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}) = E_{y,y}(-\tfrac{1}{4},\tfrac{1}{4},-\tfrac{1}{4}) = E_{y,y}(-\tfrac{1}{4},-\tfrac{1}{4},\tfrac{1}{4}) = E_{y,y}(\tfrac{1}{4},-\tfrac{1}{4},-\tfrac{1}{4})$$
$$E_{x,x}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}) = E_{y,y}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4})$$
$$E_{z,z}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}) = E_{z,z}(-\tfrac{1}{4},\tfrac{1}{4},-\tfrac{1}{4}) = E_{z,z}(-\tfrac{1}{4},-\tfrac{1}{4},\tfrac{1}{4}) = E_{z,z}(\tfrac{1}{4},-\tfrac{1}{4},-\tfrac{1}{4})$$
$$E_{x,y}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}) = -E_{x,y}(-\tfrac{1}{4},\tfrac{1}{4},-\tfrac{1}{4}) = E_{x,y}(-\tfrac{1}{4},-\tfrac{1}{4},\tfrac{1}{4}) = -E_{x,y}(\tfrac{1}{4},-\tfrac{1}{4},-\tfrac{1}{4})$$
$$E_{x,z}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}) = E_{x,z}(-\tfrac{1}{4},\tfrac{1}{4},-\tfrac{1}{4}) = -E_{x,z}(-\tfrac{1}{4},-\tfrac{1}{4},\tfrac{1}{4}) = -E_{x,z}(\tfrac{1}{4},-\tfrac{1}{4},-\tfrac{1}{4})$$
$$E_{y,z}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}) = -E_{y,z}(-\tfrac{1}{4},\tfrac{1}{4},-\tfrac{1}{4}) = -E_{y,z}(-\tfrac{1}{4},-\tfrac{1}{4},\tfrac{1}{4}) = E_{y,z}(\tfrac{1}{4},-\tfrac{1}{4},-\tfrac{1}{4})$$
$$E_{x,z}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}) = E_{y,z}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4})$$

TABLE II

$$(s/s)_{11} = E_{s,s}(0,0,0) + 2E_{s,s}(0,0,1)\cos 4\zeta$$
$$(s/x)_{11} = 0$$
$$(s/y)_{11} = 0$$
$$(s/z)_{11} = 2iE_{s,z}(0,0,1)\sin 4\zeta$$
$$(x/x)_{11} = (y/y)_{11} = E_{x,x}(0,0,0) + 2E_{x,x}(0,0,1)\cos 4\zeta$$
$$(z/z)_{11} = E_{z,z}(0,0,0) + 2E_{z,z}(0,0,1)\cos 4\zeta$$
$$(x/y)_{11} = 2iE_{x,y}(0,0,1)\sin 4\zeta$$
$$(x/z)_{11} = 0$$
$$(y/z)_{11} = 0$$
$$(s/s)_{12} = 4E_{s,s}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4})(\cos\xi\cos\eta\cos\zeta - i\sin\xi\sin\eta\sin\zeta)$$
$$(s/x)_{12} = -4E_{s,x}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4})(\cos\xi\sin\eta\sin\zeta - i\sin\xi\cos\eta\cos\zeta)$$
$$(s/y)_{12} = -E_{s,x}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4})(\sin\xi\cos\eta\sin\zeta - i\cos\xi\sin\eta\cos\zeta)$$
$$(s/z)_{12} = -4E_{s,z}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4})(\sin\xi\sin\eta\cos\zeta - i\cos\xi\cos\eta\sin\zeta)$$
$$(x/x)_{12} = (y/y)_{12} = 4E_{x,x}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4})(\cos\xi\cos\eta\cos\zeta - i\sin\xi\sin\eta\sin\zeta)$$
$$(z/z)_{12} = 4E_{z,z}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4})(\cos\xi\cos\eta\cos\zeta - i\sin\xi\sin\eta\sin\zeta)$$
$$(x/y)_{12} = -4E_{x,y}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4})(\sin\xi\sin\eta\cos\zeta - i\cos\xi\cos\eta\sin\zeta)$$
$$(x/z)_{12} = -4E_{x,z}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4})(\sin\xi\cos\eta\sin\zeta - i\cos\xi\sin\eta\cos\zeta)$$
$$(y/z)_{12} = -4E_{x,z}(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4})(\cos\xi\sin\eta\sin\zeta - i\sin\xi\cos\eta\cos\zeta)$$

the neighbouring atoms. In our Table we have only the E-interals depending on the vectors $\boldsymbol{r}_1 = (0, 0, c)$ and $\boldsymbol{t}_1 = (\frac{1}{4}a, \frac{1}{4}a, \frac{1}{4}c)$. We have the direction consinus for $\boldsymbol{r}_2 : l = 0,\ m = 0,\ n = 1$ and for

$$\boldsymbol{t}_1 : l = \alpha = a/\sqrt{2a^2 + c^2} \approx 0.683,\ m = \alpha,\ n = \beta = c/\sqrt{2a^2 + c^2} \approx 0.262.$$

Hence we can tabulate the matrix components in the two-centre approximation.

TABLE III

$$(s/s)_{11} = s_0 + 2(ss\sigma)_2 \cos 4\zeta$$
$$(s/x)_{11} = 0$$
$$(s/y)_{11} = 0$$
$$(s/z)_{11} = 2i(sp\sigma)_2 \sin 4\zeta$$
$$(x/x)_{11} = (y/y)_{11} = p_0 + 2(pp\pi)_2 \cos 4\zeta$$
$$(z/z)_{11} = p_1 + 2(pp\sigma)_2 \cos 4\zeta$$
$$(x/y)_{11} = 0$$
$$(x/z)_{11} = 0$$
$$(y/z)_{11} = 0$$
$$(s/s)_{12} = 4(ss\sigma)_1 (\cos\xi \cos\eta \cos\zeta - i \sin\xi \sin\eta \sin\zeta)$$
$$(s/x)_{12} = -4\alpha(sp\sigma)_1 (\cos\xi \sin\eta \sin\zeta - i \sin\xi \cos\eta \cos\zeta)$$
$$(s/y)_{12} = -4\alpha(sp\sigma)_1 (\sin\xi \cos\eta \sin\zeta - i \cos\xi \sin\eta \cos\zeta)$$
$$(s/z)_{12} = -4\beta(sp\sigma)_1 (\sin\xi \sin\eta \cos\zeta - i \cos\xi \cos\eta \sin\zeta)$$
$$(x/x)_{12} = (y/y)_{12} = 4[\alpha^2(pp\sigma)_1 + (1-\alpha^2)(pp\pi)_1] (\cos\xi \cos\eta \cos\zeta - i \sin\xi \sin\eta \sin\zeta)$$
$$(z/z)_{12} = 4[\beta^2(pp\sigma)_1 + (1-\beta^2)(pp\pi)_1] (\cos\xi \cos\eta \cos\zeta - i \sin\xi \sin\eta \sin\zeta)$$
$$(x/y)_{12} = -4\alpha^2[(pp\sigma)_1 - (pp\pi)_1] (\sin\xi \sin\eta \cos\zeta - i \cos\xi \cos\eta \sin\zeta)$$
$$(x/z)_{12} = -4\alpha\beta[(pp\sigma)_1 - (pp\pi)_1] (\sin\xi \cos\eta \sin\zeta - i \cos\xi \sin\eta \cos\zeta)$$
$$(y/z)_{12} = -4\alpha\beta[(pp\sigma)_1 - (pp\pi)_1] (\cos\xi \sin\eta \sin\zeta - i \sin\xi \cos\eta \cos\zeta)$$

We are left with only 11 constants.

To write the secular equation we must discuss also the overlap integrals between atomic functions $\int \varphi_m^*(\boldsymbol{r} - \boldsymbol{r}_j)\,\varphi_n(\boldsymbol{r} - \boldsymbol{r}_k)\,d\boldsymbol{r}$ because the s and p functions have a large extension in space. This problem and the solution of the secular equation will be discussed in the next paper.

I wish to thank Dr A. V. Gold for suggesting this investigation.

INSTITUTE OF PHYSICS, POLISH ACADEMY OF SCIENCES
(INSTYTUT FIZYKI, PAN)

REFERENCES

[1] J. C. Slater, G. F. Koster, Phys. Rev., **94** (1954), 1498.
[2] M. Miąsek, Acta Phys. Pol., **16** (1957), 343.
[3] — ibid. **16** (1957), 447.

BULLETIN DE L'ACADÉMIE
POLONAISE DES SCIENCES
Série des sci. math., astr.
et phys.—Vol. VIII, No. 2, 1960

On the Possibility of Neutral Currents in Four-Fermion Interactions

by

W. MAJEWSKI

Presented by L. INFELD on December 5, 1959

1. Two "anomalous" events of the decay $K^+ \to \pi^+ +$neutrals are known which energetically do not fit the schemes $K^+_{\pi 2} \to \pi^+ + \pi^0$ ($E^{\text{kin}}_{\pi^+} \approx 107.7$ MeV) or $K^+_{\pi 3} \to \pi^+ + 2\pi^0$ ($E^{\text{kin}}_{\pi^+ \max} \approx 53$ MeV): one event [1] with $E^{\text{kin}}_{\pi^+} = (60 \pm 1)$ MeV, another [2] with $E^{\text{kin}}_{\pi^+} = (61.7 \pm 1.5)$ MeV; they were found among 1400 of the ordinary $K^+_{\pi 2}$ decays. In the above mentioned papers an interpretation of these decays as $K^+ \to \pi^+ + \pi^0 + \gamma$ ($E^{\text{kin}}_{\pi^+ \max} \approx 108$ MeV) was adopted based essentially on the assumption that the energy spectrum of π^+-mesons from such decay has a wide peak just in the region of 60 MeV. But recently Hiida [3] has shown that the taking into account of the mechanism of the internal bremsstrahlung gives quite a different π^+-spectrum, without the peak and strongly increasing with energy.

This fact, in our opinion, diminishes the probability of the interpretation $K^+ \to \pi^+ + \pi^0 + \gamma$ and obliges to turn our attention to other possibilities.

2. We shall consider one of them, namely the decay

$$K^+ \to \pi^+ + \tilde{\nu} + \nu, \qquad (E^{\text{kin}}_{\pi^+ \max} \approx 127 \text{ MeV}) \tag{1}$$

allowed in the scheme of the four-fermion interactions with neutral currents (cf. [4]). As it is known, the recent successes of Fermi's theory of weak interactions with V—A coupling in explanation of the β and μ decays intensified the attempts to extend the universality of this interaction on all other weak processes, including those with a change of strangeness.

In a general case this is formulated as a self-interaction of the charged chiral current j_λ:

$$H^{\text{weak}}_{\text{int}} = \frac{G}{\sqrt{2}}\, j_\lambda\, j^+_\lambda, \qquad j_\lambda = j^L_\lambda + J_\lambda + G_\lambda,$$

where j_λ^L is the lepton current, J_λ and G_λ are the currents of strongly interacting particles conserving and non-conserving strangeness, respectively; $G/\sqrt{2}$ is the universal coupling constant. The isovector character of the current J_λ agrees with the experiment; in [5] consequences of the assumption that G_λ has the transformation properties of the isospinor were considered. The conclusions do not contradict the experiments on the decays of K-mesons and hyperons [6].

In principle there is a possibility that the neutral components of the currents J_λ and G_λ also interact. So far, a sufficiently well-grounded theoretical principle has not been proposed which would explain the empirical rule $|\Delta Q| = 1$ in fermion currents. The hypothesis of the charged intermediate vector X-meson is apparently in disagreement with the experimental ratio $w(\mu \to e + \gamma)/w(\mu \to e + \nu + \tilde{\nu})$. The assumption that there are two neutrinos with opposite lepton numbers, the one coupled to the electron and the other to the muon, forbids only currents $(\bar{\mu}e)$, $(\nu_1\ \nu_2)$ (the γ-matrices are omitted), without explaining the absence of many other possible processes (e. g. $K^\pm \to \pi^\pm + \mu^\pm + \mu^\mp$), $K^0_{1,2} \to e^+ + e^-$, etc.). Besides, the agreement between the rule $|\Delta I| = \frac{1}{2}$ for the nonleptonic decays of strange particles, on one hand, and the isospinor G_λ-current and isovector J_λ-current, on the other, would demand the inclusion of neutral couplings for these baryon currents. It is so, because to explain e.g. the decay $\Lambda \to N + \pi$ we must take the interaction $(p\Lambda)\ (\bar{n}p) + (n\Lambda)\ (nn)$ instead of only $(\bar{p}\Lambda)\ (\bar{n}p)$ [6]. These neutral baryon currents would be able to interact with the neutral lepton current. The presence in the latter of the currents $(\bar{\mu}e)$, (ee), $(\bar{\mu}\mu)$ seems now to be improbable *).

3. The hypothetical interactions with $(\nu\nu)$-current would be most easily found in the K-meson's decays. Assuming a) the isospinor transformation properties of the strangeness nonconserving current G_λ, with $|\Delta Q| = 0,1$, b) the universality of the weak V-A coupling for both charged and neutral currents, it is possible to evaluate the probability of the assumed mode (1) by comparison with the known decay $K^+_{e3} \to \pi^0 + e^+ + \nu$. The matrix element for the diagram in Fig. 1 is

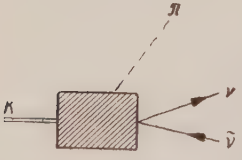


Fig. 1

$$M(K^+ \to \pi^+ + \tilde{\nu} + \nu) = \frac{G}{\sqrt{2}} [u_\nu \gamma_\lambda (1+\gamma_5) v_\nu] < \pi^+ | G_\lambda | K^+ >$$

*) Experimental evidence indicates that

$$w(\mu \to e + \gamma)/w(\mu \to e + \nu + \tilde{\nu}) < 2 \cdot 10^{-6}, \qquad w(\mu \to 3e)/w(\mu \to e + \nu + \tilde{\nu}) < 10^{-5}$$

and

$$w(\mu^- + N \to e^- + N)/w(\mu \to e + \nu + \tilde{\nu}) < 5 \cdot 10^{-4}$$

and analogously for K_{e3}^{+} with substitution π^0 for π^+ and e^+ for $\tilde{\nu}$. Neglecting the electromagnetic corrections, we have from our assumption a)

$$(2) \qquad <\pi^+|G_\lambda|K^+> = \sqrt{2}<\pi^0|G_\lambda|K^+>.$$

From the Lorentz-invariance arguments

$$<\pi^0|G_\lambda|K^+> = fK_\lambda + gL_\lambda,$$

where K_λ is 4-momentum of K-meson, L_λ — the sum of the 4-momenta of the leptons, f and g are scalar functions of $m_K^{-1} K_\lambda L_\lambda (= E_\pi^{\text{kin}}$ in the rest system of K).

Using the Dirac equations for u_e and u_ν, we obtain

$$(3) \qquad \begin{cases} M(K^+ \to \pi^+ + \tilde{\nu} + \nu) = \sqrt{2}\,\dfrac{G}{\sqrt{2}}\, u_\nu [f\gamma_\lambda K_\lambda (1+\gamma_5)]\, v_\nu, \\ M(K^+ \to \pi^0 + e^+ + \nu) = \dfrac{G}{\sqrt{2}}\, \bar{u}_\lambda [f\gamma_\lambda K_\lambda (1+\gamma_5) - im_e g(1-\gamma_5)]\, v_e. \end{cases}$$

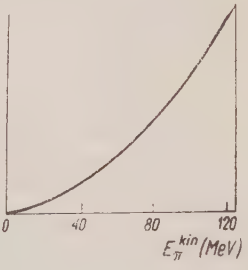
From the perturbation calculations it may be concluded that f and g are of the same order of magnitude and that they weakly depend on π-meson energy (the latter conclusion results also from the dispersion relations [7]); thus we can neglect $m_e g$ in comparison to $m_K f$. Neglecting consequently the electron mass and using (2), we come to the branching ratio

$$(4) \qquad w(K^+ \to \pi^+ + \tilde{\nu} + \nu) / w(K^+ \to \pi^0 + e^+ + \nu) \approx 2.$$

The relative frequency of the K_{e3} decay is known from experiment to be $\approx 4\%$. Then from (4) for the relative frequency of mode (1) we obtain $\approx 8\%$. This mode is experimentally most definitely distinguished from the $K_{2\pi}^{+}$, $K_{3\pi}^{+}$ modes for the π^+-meson energy in the range 55 MeV $\leqslant E_{\pi^+}^{\text{kin}} \leqslant$ 80 MeV [2]. The matrix element (3) gives the following pion spectrum

$$\frac{dw}{dE_\pi} = G^2 (2\pi)^{-3} (m_K/6) (E_\pi^2 - m_\pi^2)^{3/2} |f(E_\pi)|^2$$

represented in Fig. 2 neglecting the energetical dependence of f. In the "anomalous" region (from 55 to 80 MeV) lie 18% of the total number of decays (1). Taking into account that the relative frequency of $K_{\pi 2}^{+}$ mode is $\approx 25\%$ we obtain the theoretical estimation



Rys. 3

$$(5) \qquad w(K^+ \to \pi^+ + \tilde{\nu} + \nu)_{\text{in anom. region}} / w(K^+ \to \pi^+ + \pi^0) \approx \frac{84}{1400}.$$

Experimentally, the same ratio is $\leqslant 2/1400$, and therefore we have an obvious disagreement with (5).

Thus we conclude that, if there exists a coupling of the neutral counterpart of G_λ with the neutrino current, then the assumptions a) and b) are not applicable to it *).

The author wishes to express his thanks to Professor M. A. Markov for helpful advice during his stay at the Joint Institute for Nuclear Research, Dubna, USSR, and to B. N. Valuyev for critical discussions.

Note added in proof. T. Yamanouchi [10] suggests that events [1], [2] are the decays at rest of D-meson $D^+ \to \pi^+ + K^+$.

INSTITUTE OF PHYSICS, POLISH ACADEMY OF SCIENCES
(INSTYTUT FIZYKI, PAN)

REFERENCES

[1] G. Harris, J. Lee, J. Orear and S. Taylor, Phys. Rev., **108** (1957), 1561.
[2] D. J. Prowse and D. Evans, Nuovo Cim., **6** (1958), 856.
[3] K. Hiida, Nuovo Cim., **13** (1959), 1117.
[4] S. Oneda, Nuclear Physics, **3** (1957), 598.
[5] S. Okubo, R. E. Marshak, E. C. G. Sudarshan, W. B. Teutsch and S. Weinberg, Phys. Rev., **112** (1958), 665.
[6] R. E. Marshak, Report on 1959 Annual Intern. Conf. on High Energy Physics in Kiev.
[7] N. C. Cabibbo and R. Gatto, Nuovo Cim., **13** (1959), 1036.
[8] Crawford and others, Phys. Rev. Letters, **1** (1958), 377.
[9] J. Leitner and others, Phys. Rev. Letters, **3** (1959), 186.
[10] T. Yamanouchi, Phys. Rev. Letters, **3** (1959), 480.

*) The recent results [8], [9] on the leptonic decay modes of Λ and Σ indicate probably that four-fermion interactions responsible for them are several times weaker than interactions leading to β and μ decays; but still there may be another kind of universality between strangeness nonconserving interactions.

BULLETIN DE L'ACADÉMIE
POLONAISE DES SCIENCES
Série des sci. math., astr.
et phys.—Vol. VIII, No. 2, 1960

EXPERIMENTAL PHYSICS

Electron Beam Current Modulation by Thin Layers

by

A. FRYSZMAN

Presented by A. JABŁOŃSKI on November 18, 1959

In [1] the following expression was derived for the current passing through a surface element of a thin layer subjected to periodical bombardment with an electron beam:

$$I_s = d I_w \frac{(e^{\pm\alpha} - 1)\,(1 - e^{-\beta})}{\pm\alpha\,(1 - e^{\pm\alpha} e^{-\beta})}, \tag{1}$$

We shall use the following notation:

$$\alpha = \frac{I_w k}{C} T_k; \qquad \beta = \frac{T_0}{RC}$$

I_s — mean current through a surface element during switching,
R — transverse resistance of a surface element,
k — tangent of angle α of the slope of secondary emission linearized characteristic (Fig. 1),
T_k — time of switching of surface by the beam,
T_0 — time between switching periods,
U — voltage on layer with respect to conducting substratum,
I_w — current intensity of electron beam,
U_{ps} — voltage on conducting substratum with respect to cathode of electron gun,
U_{ac} — voltage accelerating electron beam.

Depending on the choice of the region of the secondary emission characteristic within which the layer is functioning, two values of the coefficient k and, hence, of α, are possible. The sign of α must be positive if, when the layer is charged, an increase in potential of the surface with respect to the substratum produces an increase in the current, and negative, if an increase in potential on the layer brings about a fall in current intensity. The value of the coefficient d is determined in the case of

linearization of the secondary emission characteristic according to the equation of the straight line as follows [1]:

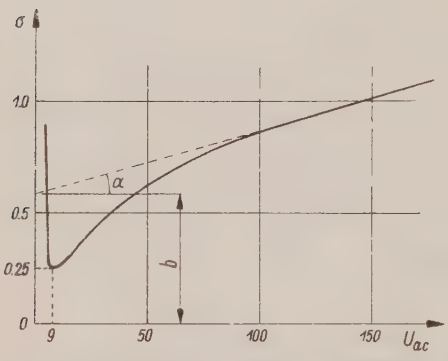


Fig. 1. Secondary emission *versus* accelerating voltage in Sb_2S_3

$$\sigma = d \pm kU$$

for $\sigma < 1$

$$d = 1 - kU_{ps} - b$$

and for $\sigma > 1$,

$$d = kU_{ps} \pm b - 1 \qquad \text{(see Fig. 1).}$$

From the possible difference in sign it is seen that two different mechanisms of functioning of a thin layer subjected to periodical bombardment by an electron beam can exist.

The first of these mechanisms is related to values of the parameters $k > 0$, $\sigma < 1$ or $k > 0$ and $\sigma > 1$, the second to — $k < 0$, $\sigma > 1$ or $k < 0$ and $\sigma < 1$.

In the sequel the sign $\pm$ will be used for α, if the kind of functioning is not specifically indicated.

It is the aim of the present paper to give an analysis of the working mechanism of a thin layer when bombarded periodically with an electron beam. As the equations are derived for a surface element functioning independently of the whole layer, the equation and analysis are applicable also to layers exhibiting a mosaic pattern.

Eq. (1) determines the parameters of the layer admitting of the establishment of a steady state of equilibrium as between the process of charging of the layer during switching and that of discharging during the period between two switchings.

Variation of any one of the parameters C, R, I_w, k, T_k, U_{ps}, T_0 produces a transition from one steady state to another characterized by a different current intensity through the layer and another voltage.

Transition from one state to another is accompanied by inertance. During several (or more) switchings of the element, new values of the current and voltage become established, whereas the old values gradually disappear. The ratio of the voltage during any n-th period of the transition to that existing on the layer surface previous to the beginning of the transition determines the "rate of decay" Y_1 of the voltage, assuming the voltage on the layer to be 0 at the and of the transition. By Eq. (9) [1]:

$$Y_1 = (e^{\pm\alpha} \cdot e^{-\beta})^n. \tag{2}$$

The rate of establishment Y_2 of the new state can be determined as the ratio of the voltage on the layer at any n-th period of the transitory

state to the voltage at any period of the subsequent steady state, assuming the initial voltage on the layer as 0:

(3) $$Y_2 = 1 - (e^{\pm\alpha} \cdot e^{-\beta})^{n+1}.$$

Eqs. (1), (2) and (3) yield the inertance, sensitivity, current and other characteristics of thin layers.

1. Case of $k < 0$, $\sigma < 1$ or $k > 0$ and $\sigma > 1$. Here, the equation of the current assumes the form:

(4) $$I_s = d I_w \frac{(1 - e^{-\alpha})(1 - e^{-\beta})}{\alpha (1 - e^{-\alpha} \cdot e^{-\beta})}.$$

To give an analysis of the variations in the current through the layer, as resistance varies in photosensitive layers, the equation can be separated into a part dependent on R and describing the sensitivity:

(5) $$\varphi(R) = \frac{1 - e^{-\beta}}{1 - e^{-\alpha} \cdot e^{-\beta}}$$

and a part independent of R and accounting for the current intensity:

(6) $$A = \frac{1 - e^{-\alpha}}{\alpha}.$$

Fig. 2 brings the dependence $\varphi(R) = \varphi(1/\beta)$ for several values of the parameter $1/\alpha = C/I_w kT_k$.

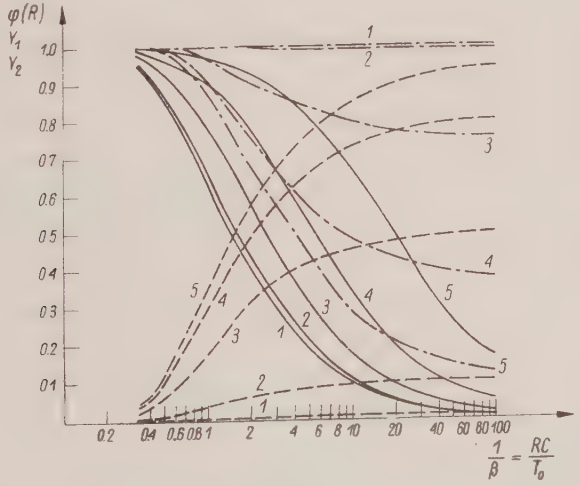


Fig. 2. Sensitivity and inertance *versus* RC of surface element:
—— sensitivity of layer, · - - - - - - - inertance Y_1, . —·—·— inertance Y_2.
Values of parameter $\frac{1}{\alpha} = \frac{C}{I_w kT_k}$:

1) $\frac{1}{\alpha} = 0.144$, 2) $\frac{1}{\alpha} = 0.435$, 3) $\frac{1}{\alpha} = 1.44$, 4) $\frac{1}{\alpha} = 4.49$, 5) $\frac{1}{\alpha} = 19.6$.

In Fig. 2, the dashed line shows the dependence of the inertance Y and the dotted line — that of the inertance Y_2, for the same values of $1/\alpha$. From Fig. 2 a decrease in inertance of the layer is seen to be achieved by a reduction in the C value of the surface element.

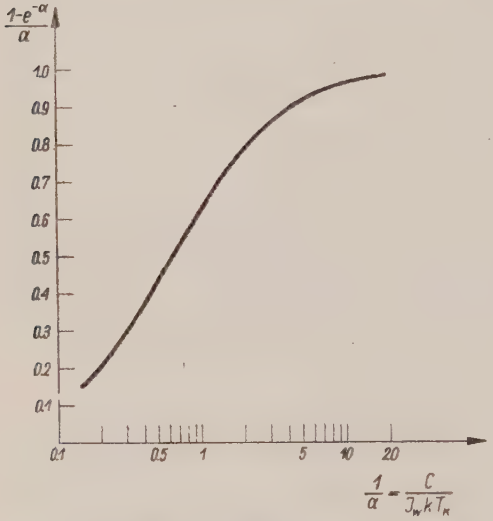


Fig. 3. Current intensity through surface element *versus* the capacitance.

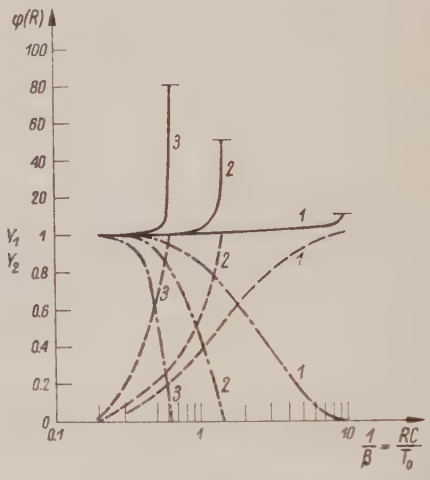


Fig. 4. Sensitivity and inertance *versus* RC of surface element:

——— sensitivity of layer,
- - - - - - inertance Y_1,
— · — · — inertance Y_2.

Values of parameter $\frac{1}{\alpha} = \frac{C}{I_w kT_k}$:

1) $\frac{1}{\alpha} = 10.4$, 2) $\frac{1}{\alpha} = 1.44$, 3) $\frac{1}{\alpha} = 0.62$.

The problem of reducing the inertance is one of importance in pick-up tubes. A reduction in the capacitance, even if RC — the sensitivity of the layer — remains unchanged, brings about a decrease in the total current through the layer. Fig. 3 shows the current intesity (6) in relative units "A" *versus* the surface element capacitance.

2. Case of $k>0$, $\sigma<1$ and $k>0$, $\sigma>1$.

Here, the equation of the current has the form:

$$I_s = dI_w \frac{(e^{\alpha}-1)(1-e^{-\beta})}{\alpha(1-e^{\alpha}\cdot e^{-\beta})}. \tag{7}$$

As above, the sensitivity of the layer

$$\varphi(R) = \frac{1-e^{-\beta}}{(1-e^{\alpha}\cdot e^{-\beta})} \tag{8}$$

and the current intensity

$$A = \frac{e^{\alpha}-1}{\alpha} \tag{9}$$

will be examined separately.

In Fig. 4, the continuous line serves to indicate the sensitivity of the layer as *versus* $1/\beta = RC/T_0$, whereas the dashed and dotted lines are those of the inertances Y_1 and Y_2, respectively, at various values of $1/\alpha = C/I_w k T_k$. As distinct from the former case, an increase in the resistance is related to a rise, not to a fall, in the current through the layer. When the pick-up tube is functioning within this region, the picture obtained resembles a photographical negative. The region of the characteristic corresponding to correct functioning of the layer is limited by the condition $e^{\alpha} \cdot e^{-\beta} < 1$, as shown by horizontal dashes in Fig. 4. Outside this boundary, the layer will exhibit durable memory: the initial voltage will increase each time as the beam switches the layer, until a voltage is attained for which $\sigma = 1$. The dependence of the current intensity on the parameter α is shown in Fig. 5.

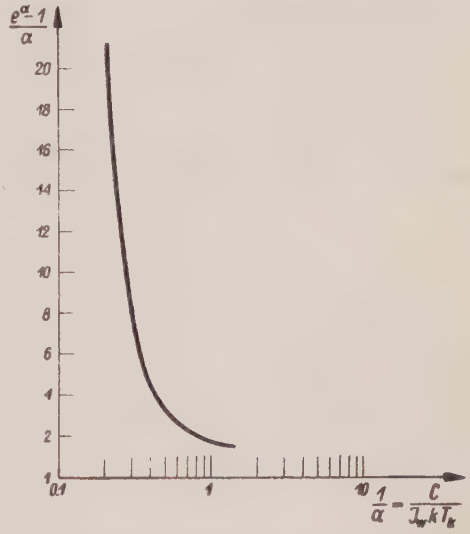


Fig. 5. Current intensity through surface element *versus* the capacitance.

The foregoing theory accounts for the effects occurring in pick-up tubes, namely that of negative picture and that of tube memory. With the formulae and curves, the parameters required for correct functioning of thin layers can be determined, viz., the resistance, capacitance, photosensitivity and inertance, and the currents through the layer computed.

Moreover, the theory accounts for prediction of current modulation by a thin layer of variable capacitance.

For this, the notation

$$e^{-\beta} = e^{-\frac{T_0}{RC}} = e^{\frac{T_0}{T_k I_w R k} \cdot \frac{I_w k}{c} T_k} = e^{-a\alpha}$$

is introduced. The current *versus* capacitance dependence in case 1 is then given by

$$I_s = dI_w \frac{(1 - e^{-\alpha})(1 - e^{-a\alpha})}{\left|1 - e^{-(a+1)\alpha}\right| \alpha} \tag{10}$$

and, in case 2, by

$$I_s = dI_w \frac{(e^{\alpha} - 1)(1 - e^{-a\alpha})}{\left|1 - e^{-(a+1)\alpha}\right| \alpha} \tag{11}$$

with the parameter a equalling

$$a = \frac{T_0}{I_w k T_k R}.$$

The author wishes to thank Professor A. Jabłoński for his discussions of the problems related to the present investigation.

INDUSTRIAL INSTITUTE OF ELECTRONICS
(PRZEMYSŁOWY INSTYTUT ELEKTRONIKI)

REFERENCES

[1] A. Fryszman, *On the working mechanism of thin layers in pick-up tubes*, Bull. Acad. Polon. Sci., Sér. sci. math. astr. et phys., **7** (1959), 733.

BULLETIN DE L'ACADÉMIE
POLONAISE DES SCIENCES
Série des sci. math., astr.
et phys.—Vol. VIII, No. 2, 1960

EXPERIMENTAL PHYSICS

Density of States Effective Mass of Electrons in InSb

by

Z. KOPEĆ

Presented by A. SOŁTAN on November 23, 1959

The results of investigations on the thermoelectromotive force in a number of semi-conductors have contributed essentially to establishing the concept of "classical" effective mass, i.e. a mass of constant value and identical for all carriers of one and the same sign in a given sample.

However, the foregoing properties of the effective mass derive from some special assumptions as to the form of dependence of the energy of the carrier on the wave vector; indeed, they correspond to a spherical, quadratic function.

Hence, accurate measurements of the thermoelectromotive force should reveal a variable value of m_{eff} not only in the case of anisotropy of the surfaces of equal energy in $\vec{k}$ space (as in Ge), but also in that of non-applicability of the quadratic approximation.

In InSb samples whose electron content is even quite insignificant, levels far remote from the extremity of the band are occupied. Hence, such material should particularly be expected to exhibit considerable divergencies from the quadratic dispersion formula.

The measurements of the thermoelectromotive force, conductivity, and Hall's constant carried out with the highest possible degree of accuracy in the course of the present investigation not only proved the non-applicability of the classical concept of effective mass, but, moreover, revealed some regularities in agreement with a generalized concept of effective mass, free of the above mentioned assumptions of sphericality and quadraticity of the dispersion formula.

The first of measurements was carried out on a number of InSb samples of high electron concentration ($\approx 9 \times 10^{18}$ cm^{-3}). From these measurements, the effective mass of the electrons was evaluated at $\mathfrak{M} \leqslant 0.041\ m_0$. Closer analysis of the experimental data obtained with one and the same sample at various temperatures pointed to an indubitable, though not great increase in $\mathfrak{M}$ with the temperature throughout the range of $125°\text{K} < T < 500°\text{K}$.

To explain this dependence the following supposition was enounced: since the transport phenomena investigated are accounted for chiefly by electrons occupying levels deeper and deeper within the conductivity band as the temperature rises, i.e. as the Fermi level is displaced "upwards", the $\mathfrak{M}$ (T) dependence indicates that the effective mass increases with the distance from the bottom of the band.

In order to check the foregoing interpretation, a number of samples of electron concentrations ranging from $2\times10^{14}\,\mathrm{cm}^{-3}$ to $9\times10^{18}\,\mathrm{cm}^{-3}$ were investigated at room temperature (which admits of the highest degree of accuracy). The hypothesis proposed predicted an increase in the effective mass of the electrons towards higher electron concentrations.

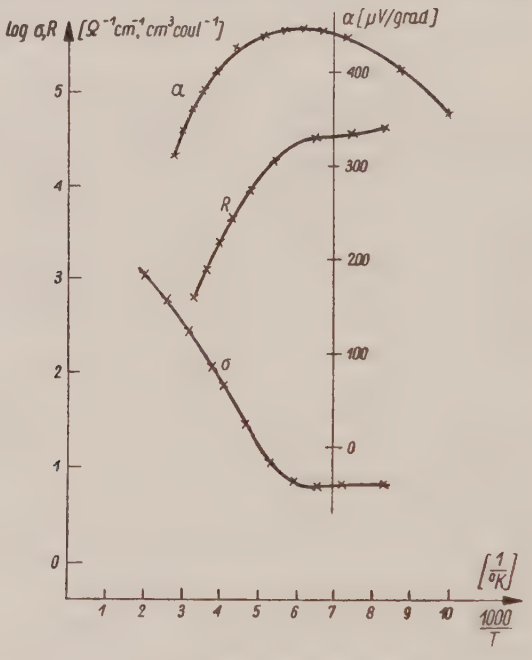


Fig. 1. Pure sample InSb

The device for measuring the thermoelectromotive force α consisted of two copper blocks having polished terminals with thermocouples welded in the vicinity of the points at which the sample contacted the blocks. In these conditions the error in measuring α did not exceed 4%. Figs. 1 and 2 bring the results of measurements of the thermoelectric force α, Hall's constant R, and the electric conductivity σ for samples presenting the extreme values of electron concentration.

As proved in [3], the transport phenomena in the case of a non-quadratic dispersion formula can be accounted for by formulae analogical to the "classical" one replacing the constant effective mass by a variable coefficient depending on the dispersion law and the kind of phenomenon under consideration. Moreover, these coefficients can vary with the carrier

concentration and the temperature. It should be stressed that the formulae preserve the "classical" form (i.e. the form corresponding to the "classical" mass).

Thus, in analyzing the experimental data, relationships formally identical with the old ones can be used; albeit, in the new relationships, the concept of effective mass assumes a more extensible connotation (that of a mass coefficient) adapted to the description of the phenomena as dependent on the real, not simplified, energy structure of the sample.

The concept of mass coefficients, when used appropriately, has been in a number of cases found to account quantitatively and univocally for experimentally detected regularities and to resolve divergencies between

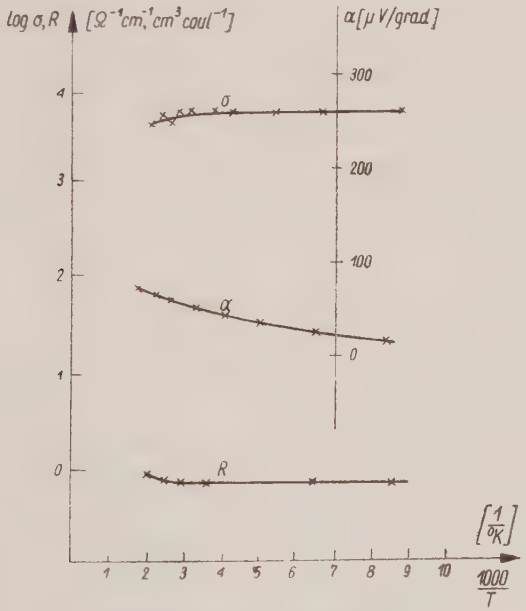


Fig. 2. Heavily dopped sample InSb

theoretical and experimental results. Among others, this has been the case of the thermoelectromotive force in InSb.

In determining the value of the thermoelectric effect, [5], the mass coefficient $\mathfrak{M}$ in formula (1) is decisive:

$$\alpha = \frac{k}{e}[A - \eta(\mathfrak{M})], \tag{1}$$

the function $\eta(\mathfrak{M})$ is determined by the relation

$$n = \frac{2(2\pi\mathfrak{M}kT)^{3/2}}{h^3}\frac{2}{\sqrt{\pi}}F_{1/2}(\eta). \tag{2}$$

Here, k denotes Boltzmann's constant, e the elementary charge, h Planck's

constant, n the concentration of the electrons, A the average energy of the electrons in kT units, $F_{1/2}(\eta)$ Fermi's integral defined as

$$F_{1/2}(\eta) = \int_0^\infty \frac{x^{1/2}}{1 + \exp(x - \eta)}\, dx,$$

whereas η in the Fermi level energy (referred to the bottom of the band) in kT units.

Some degree of error caused by the inexact knowledge of η (Hall's effect and electric conductivity) and the error inherent in the evaluation of the constant A are attached to the value of $\mathfrak{M}$ calculated from α (Eq. 1). Since the value of A is identical for various concentrations within the framework of one and the same scattering mechanism, the respective error can, at the worst, lead to a systematical inaccuracy in the absolute value of the mass computed, so that the final result will partake of the nature of an evaluation. It will have almost no effect upon the shape of the dependence of $\mathfrak{M}$ on the concentration, as observed.

We shall refrain from entering upon the details of the scattering process, in itself one of considerable interest. Here, only the results of computations carried out for the following scattering mechanisms will be presented:

I. Phonon-acoustical scattering, not accounting for the correction for the divergence from quadratic band structure;

II. Phonon-acoustical scattering, accounting for the correction for divergence from quadratic band structure;

III. Phonon-optical scattering, accounting for divergence from parabolical structure according to Ehrenreich's computations [1].

In Fig. 3, the results of this analysis of the experimental data are presented, and are compared with the theoretical curve derived from the generalized effective mass method. The computations are based on the dispersion formula for the conductivity band in InSb derived recently along theoretical lines by E. O. Kane [2].

Agreement between the experimental and theoretical data is seen to be good in each distinct case of the scattering mechanism if account be taken of the complexity of the present analysis; albeit, it is not possible to decide, whether scattering on optical vibrations plays an essential part. as neither curve II nor curve III coincide with the theoretical curve well enough to rule out the rivalling mechanism. The scattering mechanism in InSb is at present the object of studies along different lines [1], [4].

Thermoelectromotive force measurements are found to be insufficiently sensitive to the mechanism to decide the problem in the existing conditions of precision of both theory and experiment. The problem is more easily tackled from the direction of some other electronic processes. On the other hand, the rapid rise in the effective mass value with increasing concentrations is independent of the assumptions made as to scattering.

This rapid rise has been proved to be in agreement with theoretical predictions based on computation.

It should be stressed that never in the course of the present analysis have numerical values of a parameter been adjusted to agree with the experimental data obtained.

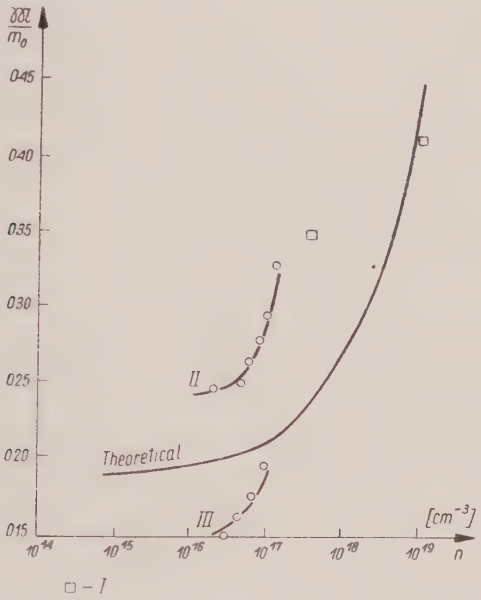


Fig. 3

The present investigation was undertaken and, to a large extent, carried out with the direct help of Professor Dr A. F. Joffe, to whom the author wishes to express his most cordial indebtedness. Moreover, the author wishes to stress the very friendly reception he was given in Professor Dr Joffe's laboratory on the part of the leading staff, and, more especially, on that of Dr I. V. Motchan and Dr A. J. Blum.

The author is also indebted to Professor Dr L. Sosnowski for his valuable discussions, to Mrs. W. Szymańska and Dr M. Szymchowicz for taking part in some of the experiments, and to Mr. J. Giriat for providing him with high purity InSb samples of his own production.

INSTITUTE OF PHYSICS, POLISH ACADEMY OF SCIENCES
(INSTYTUT FIZYKI, PAN)

REFERENCES

[1] H. Ehrenreich, J. Phys. Chem. Solids, **2** (1957), 131.
[2] E. O. Kane, J. Phys. Chem. Solids, **1** (1957), 122.
[3] Z. Kopeć, Acta Phys. Pol. (in press).
[4] M. Rodot, J. Phys. Chem. Solids, **8** (1959), 358.
[5] J. Tauc, Czech. J. Phys., **3** (1953), 120.

BULLETIN DE L'ACADÉMIE
POLONAISE DES SCIENCES
Série des sci. math., astr.
et phys.—Vol. VIII, No. 2, 1960

EXPERIMENTAL PHYSICS

On the Scattering of Electrons in InSb-*n*

by

Z. KOPEĆ

Presented by A. SOŁTAN on November 23, 1959

The scattering of electrons in InSb is a problem still subject to discussion. The semi-conductor is of the diamond type structure similar to germanium: hence, the phonons of the acoustical branch might be expected to account for scattering of the free carriers. On the other hand, the presence at the lattice points in InSb of two different types of atoms belonging to two different columns of the periodic system introduces an admixture of ion bond lending greater importance to the scattering mechanism on the optical branch of vibrations in a lattice that, in the present case, is of a partially polar character. However, first evaluations point to a rather unimportant role of scattering on optical phonons at room temperature, when the vibration spectrum is poor in (high energy) optical phonons.

Nevertheless, some authors propose to accept scattering on optical phonons as the dominant mechanism, since their computations based on the assumption of a phonon-acoustical mechanism lead to disagreement with experimental data. Thus, K. W. Keyes [4] and H. Ehrenreich [2] conclude that the mobility computed from the well known Bardeen-Shockely formula [1], which deals with scattering on acoustical phonons, is one order of magnitude higher than the experimental value. The divergence is, indeed, removed, if scattering on optical phonons is assumed to be the dominant effect, as the implication of additional action on the electrons by a sufficiently strong perturbing potential of a partly polar nature can reduce considerably the value of the mobility predicted. To obtain agreement of the theoretical and experimental values, however, it is necessary to introduce some assumptions as to factors, the independent determination of which presents great difficulties, namely: the rate of ionicity and concentration of the optical phonons. Such a procedure is of a nature to rest on the adjustement of at least one numerical coefficient. It is here that the essential weakness of this type of explanation of the anomally low electron mobility in InSb resides.

The present paper is aimed at proving that the introduction of an electron effective mass as corrected according to the present author's analysis of the case of non-quadratic dispersion formula [5] admits of an explanation of the experimental mobility values and of a number of more subtle regularities presented by the latter, quite independently of the controversial hypothesis of scattering on optical phonons.

Our analysis is based on the statement that the usually accepted approximation of the parabolical dependence of the electron energy on the wave vector k is not applicable to InSb [4]. Indeed, the approximation accounts adequately for the bottom of the band only, whereas, because of the low density of states a considerable portion of the band is occupied in the case of InSb, even at low electron concentrations. Rejection of the assumption of parabolicity leads to the conclusion that the various electronic phenomena should be described with the use of various effective mass quantities (representing a generalization of the unique effective mass appearing in the simplest parabolical scheme), which can be computed univocally if the dispersion formula for the band is known.

Let us consider the electron mobility formula in the case of scattering on acoustical phonons:

$$\mu_{ph} = \frac{(8\pi)^{1/2}\, e\hbar \cdot C_{ii}}{3\, m_\mu^{5/2} (kT)^{3/2} E_1^2}. \tag{1}$$

Now, according to the author's analysis, the effective mass in Eq. (1), m_μ, differs from the mass m_c obtained from the cyclotron effect. All the other quantites have the same meaning as in Shockley's "classical" expression, which is formally identical with Eq. (1); thus, e denotes the elementary charge, $2\pi\hbar$ — Planck's constant, C_{ii} — the elasticity constant, k — Boltzmann's constant, T — the Kelvin temperature, and E_1 — the displacement of the bottom of the conductivity band per unit lattice dilatation. It is seen from the Table (which is a reproduction of Table II

TABLE

Theoretical values of m_μ

T \ n	10^{15}	10^{16}	10^{17}	10^{18}	10^{19}
50° K	.0156	.018	.025	.048	.13
100° K	.0165	.018	.023	.047	.11
300° K	.0193	.025	.029	.042	.09

of [5]), that in InSb samples of the highest degree of purity (at room temperature $N_D - N_A = 2 \times 10^{16}$ cm^{-3}), m_μ is about the double of m_c. If this value is taken to replace m_c a value of μ_{ph} that is six times smaller is obtained, thus yielding a value that is nearer the experimental one. It should also be taken into account that the value of μ_{ph} as computed by

Keyes is most probably too high also because of his assumption of a value of E_1 corresponding to germanium. The value of E_1 is not easily determined experimentally. However, Keyes' assumption does not seem to be fully justified in the light of certain indirect experimental data. The fact that the width of the forbidden band is more susceptible to temperature in the case of InSb than in that of Ge should point to a much higher value of E_1 in the former. A value of E_1 in InSb three times that in Ge would yield mobility in agreement with experiment.

Consistent consideration of the numerical results of [5], as summarized in the Table, admits of an explanation, by scattering on acoustical phonons, of various other important experimental facts which hitherto failed to be properly appraised, namely: the concentrational dependence of electron mobility in InSb, and the concentrational dependence of the mobility *versus* temperature curves.

From the Table m_μ is seen to rise rapidly with the electron concentration, at a given temperature.

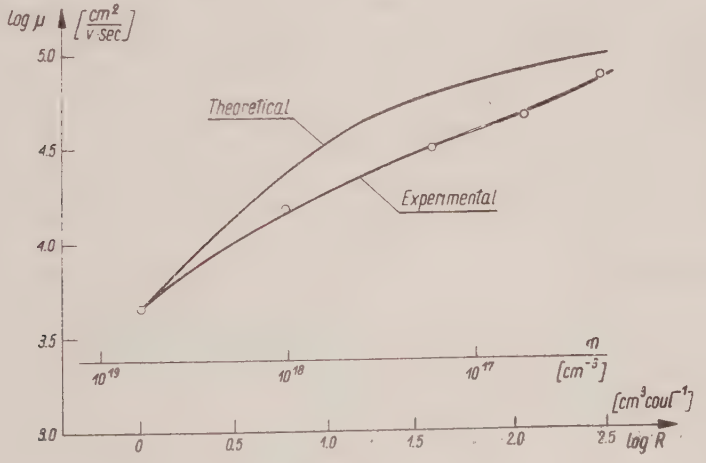


Fig. 1

At 300°K, as the concentration changes form 10^{16} cm^{-3} to 10^{19} cm^{-3}, the mass increases from 0.0255 m_0 to 0.09 m_0. Such an increase in the value of the mass should lead to a mobility 25 times smaller. In reality, this conclusion is fully corroborated by experiment. InSb samples of the highest degree of purity exhibit an electron mobility of $\mu \approx 80{,}000$ cm^2/V.sec, whereas those of very high admixture content yield a mobility value μ, of less than μ 4,000 cm V. sec. Experimental and theoretical data are assembled in Fig. 1 An alternative explanation on the basis of ionic scattering at high concentrations of the admixtures should be rejected, since, in reality, a fall in electron mobility in InSb is observed with the rise of temperature, whereas in the case of an ionic mechanism an increase should be expected to occur.

Yet another experimental argument in favour of the electron scattering mechanism in InSb arises from a comparison of the mobility *versus* temperature curves in a sample of very low and in one of very high carrier concentration.

From the Table m_μ is seen to vary in the opposite sense in either case: in pure samples the effective mass m_μ increases with the temperature, whence an all-over temperature dependence of μ should be expected of the form $\mu \sim T^{-s}$, with $s > 1.5$, which is stronger than in the classical case corresponding to $s = 1.5$ — in reality experiment yields $s = 1.6 - 1.7$. On the contrary; in doped samples m_μ decreases with the temperature. This should lead to a lessening of temperature dependence of the mobility with respect to the classical case. In reality, experiment proves that doped samples exhibit a characteristically weak temperature dependence of the mobility, with a value of $s < 1.5$ in agreement with our prediction.

The fact that an explanation for both the concentrational dependence of μ — an entirely incomprehensible phenomenon in the case of scattering on the optical branch — and the so very specific and involved temperature dependence of μ provided on the basis of the "phonon-acoustical mechanism" would seem to point to electron scattering chiefly on acoustical phonons in InSb at room temperature.

INSTITUTE OF PHYSICS, POLISH ACADEMY OF SCIENCES
(INSTYTUT FIZYKI, PAN)

REFERENCES

[1] J. Bardeen, W. Shockley, Phys. Rev., **80** (1950), 72.
[2] H. Ehrenreich, J. Phys. Chem. Solids, **2** (1957), 131.
[3] E. O. Kane, J. Phys. Chem. Solids., **1** (1957), 122.
[4] A. W. Keyes, Phys. Rev., **99** (1955), 490.
[5] Z. Kopeć, Acta Phys. Polon. (in press).

BULLETIN DE L'ACADÉMIE
POLONAISE DES SCIENCES
Série des sci. math., astr.
et phys.—Vol. VIII, No. 2, 1960

Field Homogenizing Iron Plates for Nuclear Spin Resonance Spectrometer

by

K. ANTONOWICZ

Presented by A. JABŁOŃSKI on December 1, 1959

Nuclear resonance spectrographs of high resolving power require a magnetic field, the homogeenity of which is of the order of 10^{-7}—10^{-8} within a volume of several mm^3. Such a homogeneity within a greater volume would be of course desirable. Since the construction of an electromagnet of such high resolving power presents considerable difficulties, various methods are adopted for improving the homogeneity of its field. Apart from rotation of the sample (which method, indeed, does not alter the homogeneity of the field), ring shims [1], or pairs of coils compensating field inhomogeneity [2], or more complicated coil systems can be used [3].

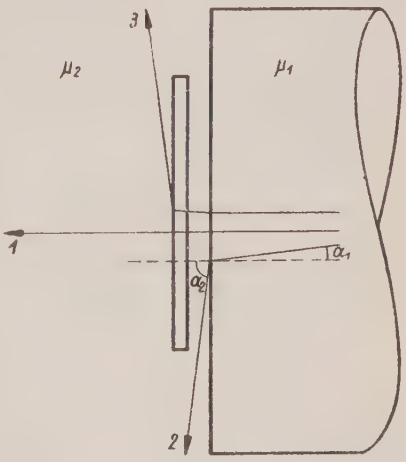


Fig. 1

A different method of improving the field homogeneity, being a modification of that described by E. M. Purcell [4], was used in the construction of the high resolution nuclear resonance spectrograph in this laboratory. Purcell proposed pole caps laminated transversally to the field direction, the effect of which resembles that of a magnetic filter. In the present case, the pole caps were made of one piece of iron each, whilst the filter consisted of two symmetrically disposed plates of soft iron parallel to the pole surfaces. The effect of the filter can be explained by considering the laws of refraction of the magnetic force lines at the iron air boundary. In Fig. 1 the passage of three force lines through one of the filter plates is shown. The magnetic force line 1 perpendicular to the pole surface is not deflected. Line 2,

incident at a large angle, is strongly deflected at angle α_2 according to the relation

$$\frac{\operatorname{tg} \alpha_1}{\operatorname{tg} \alpha_2} = \frac{\mu_2}{\mu_1},$$

where μ_1 and μ_2 denote the magnetic permeabilities of iron and air, respectively. Line 3, which is almost parallel to line 1, is slightly deflected at the pole surface and considerably at the plane. Thus, the magnetic filter transmits practically only a parallel beam of magnetic lines.

The iron (Armco) plates were made to be 120 mm in diameter and 4 mm thick, with carefully polished surfaces. A set of three screws provided for exact adjustement of the plates. Finally, the plates were set parallel to each other while, simultaneously, the line width of nuclear resonance of protons was under observation.

The effectiveness of the filter was proved by observation of the splitting of the methyl alcohol line. A sample of 100 mm^3 volume placed in the field of the electromagnet without filter yielded the simple alcohol triplet. The homogeneity of the magnetic field amounted to 10^{-6}. On introducing the magnetic filters, the lines of the CH_3 and CH_2 groups were split into a triplet and a quadruplet. The homogeneity of the field rose to 10^{-7}. In the above investigations, linear inhomogeneities of the field were not eliminated by rotation of the sample. Preliminary measurements show the magnetic field to be highly homogeneous over a considerable volume. Precise mapping of the field is in progress.

DEPARTMENT OF PHYSICS, NICHOLAS COPERNICUS UNIVERSITY, TORUŃ
(KATEDRA FIZYKI DOŚWIADCZALNEJ, UNIWERSYTET MIKOŁAJA KOPERNIKA, TORUŃ)

REFERENCES:

[1] E. R. Andrew and F. A. Ruschworth, Proc. Phys. Soc. B, 65 (1952), 301.
[2] H. Primas and H. Günthard, Helv. Phys. Acta, 30 (1957), 331.
[3] J. E. Goley, Rev. Sci. Instr., 29 (1958), 313.
[4] E. M. Purcell, N. Cim. Suppl., 6 (1957), 1234.

BULLETIN DE L'ACADÉMIE
POLONAISE DES SCIENCES
Série des sci. math., astr.
et phys.—Vol. VIII, No. 2, 1960

EXPERIMENTAL PHYSICS

Potential Scattering of Neutrons for Fe, Co, Ni, Cu, Zn, Se

by

W. RATYŃSKI, J. TURKIEWICZ and P. ŻUPRAŃSKI

Presented by A. SOŁTAN on December 1, 1959

In [1] we developed a method for the estimation of potential scattering cross-sections. This method was based on Seth's paper [2]. We measured potential scattering cross-sections σp for Al, Ag and Bi. In the present work we have extended our measurements to elements of the medium atomic weight region.

The experimental arrangement for transmission measurements and the method of data analysis were the same as in [1].

Samples of natural isotropic abundance were used in each of the transmission measurements.

Calculated cross-sections of investigated elements and corresponding effective nuclear radii are given in Table I.

TABLE I

Element	σp (barns)	R (fermis)
Fe	8.7 ± 0.3	8.5 ± 0.1
Co	4.9 ± 0.2	6.2 ± 0.1
Ni	3.9 ± 0.4	5.6 ± 0.2
Cu	6.6 ± 0.2	7.2 ± 0.1
Zn	6.6 ± 0.2	7.2 ± 0.1
Se	9.2 ± 0.6	8.5 ± 0.3

The results may be compared with the predictions of the cloudy crystal ball model [3] — [5] which gives the dependence of effective nuclear radii on atomic mass number A. Theoretical curves together with our results are shown in Fig. 1.

It is clearly seen that these results are not in good agreement with the optical model for spherical nuclei. Similiar disagreement exists in the works of other authors (cf. [6]). Particularly large discrepancies exist in the regions of atomic weights $30 < A < 90$ and $130 < A < 190$.

For heavy nuclei the strong deformation of nuclear surface must be taken into account. All the experimental data are in quite good agreement with the calculation of Chase et al. [5] involving these deformations.

Until recently it has been accepted [7] that nuclei of atomic weights $30 < A < 90$ have spherical shape. However, the data of nuclear spectro-

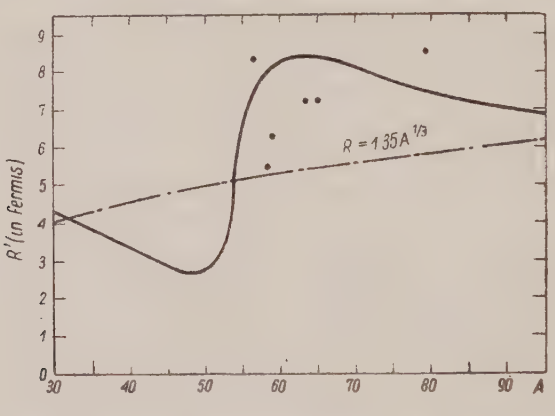


Fig. 1. The solid curve represents the predictions of the spherical optical model with diffuse surface, due to Feshbach, Porter and Camphell [6].

scopy [8] and the results of measurements of $\Gamma_n^0 D$ [2] indicate the existence of nucler deformations also in this region of atomic mass number.

The effective nuclear radii measurements, both ours and of other authors, also seem to indicate the existence of such deformations.

INSTITUTE OF EXPERIMENTAL PHYSICS, WARSAW UNIVERSITY
(INSTYTUT FIZYKI DOŚWIADCZALNEJ, UNIWERSYTET WARSZAWSKI)
INSTITUTE FOR NUCLEAR RESEARCH, POLISH ACADEMY OF SCIENCES
(INSTYTUT BADAŃ JĄDROWYCH, PAN)

REFERENCES:

[1] W. Ratyński, J. Turkiewicz, P. Żuprański, Bull. Acad. Polon. Sci., Sér. sci. math. astr. et phys., **7** (1959), 527.

[2] K. K. Seth, Proc. Gatlinburg Conference of Neutron Physics 1956, ORNL 2309, p. 5.

[3] H. Feshbach, C. E. Porter, V. Weisskopf, unpublished.

[4] V. Weisskopf, Physica, **18** (1956), 952.

[5] D. M. Chase, L. Wilets, A. R. Edmonds, Phys. Rev., **110** (1958), 1080.

[6] K. K. Seth, Rev. Mod. Phys. **30** (1958), 442.

[7] *Deformation of atomic nuclei* (in Russian) Editor L. Sliv. Moscow, 1958.

[8] A. S. Davydov, Izviestia Akad. Nauk USSR XXIII, (59), 792.

BULLETIN DE L'ACADÉMIE
POLONAISE DES SCIENCES
Série des sci. math., astr.
et phys.—Vol. VIII, No. 2, 1960

EXPERIMENTAL PHYSICS

Influence of Temperature on the Pressure Broadening of the Mercury Resonance Line

by

T. SKALIŃSKI

Presented by A. JABŁOŃSKI on December 7, 1959

Introduction

The theories of the pressure broadening of spectral lines are developed from two, essentially different, points of view *).

The classical treatement due to Lorentz, Lenz and Weisskopff (L. L. W.) considers the process of collision between the perturbing atoms of the foreign gas (P) and the absorbing (or emitting) atom (S) as fundamental, the atom S being considered as an isotropic classical oscillator. The collision causes the interruption (Lorentz) or change in phase (Lenz) of the emmitted wave train. The Fourier analysis of such a wave train gives the dispersion distribution in intensity with a half width $\gamma/2 = 1/\tau_0$; τ_0 — the time between two successive collisions — is given by the kinetic theory as:

$$\frac{1}{\tau_0} = \pi \varrho_0^2 v n_1,$$

where $\pi\varrho_0^2$ — the collision cross-section of S, v — relative mean velocity of atoms S and P, n_1 — the number of S atoms in 1 ccm.

In the case of absorption in a thick layer the total quantity of light A absorbed from a continuous background is given by:

$$A = \sqrt{\frac{\pi^2 le^2 n_1 \gamma}{mc}},$$

(l — the thickness of the layer). In the simple Lorentz theory $\gamma \sim 1/\tau_0 \sim v \sim \sqrt{T}$, whereas after Lenz-Weisskopff, the collision diameter depending on $\overline{v}\left(\varrho_0 = \sqrt[5]{\frac{k}{v}}\right)$ which gives $\gamma \sim T^{3/10}$.

*) A very complete index of references concerning the different theoretical and experimental papers on this subject may be found for inst. in [1].

Hence, one obtains for the temperature dependence of the total absorption A in the broadened line:

$$A \sim T^{1/4} \text{ according to the Lorentz theory}$$

or

$$A \sim T^{3/20} \text{ according to the Lorentz-Weisskopff theory.}$$

The introduction of potential curves visualizing the interaction in the system composed of the absorbing atom and the perturbing one, brought an entirely new point of view in the treatment of the pressure broadening. In such a case, in conformity with the Franck-Condon principle applied for the transition of the perturbed atom S, the line shape is conditioned by two factors: 1) by the form of the potential curve for the system $S - P$; 2) by the occurrence distribution for different possible configurations of S and P.

Supposing that the interaction potential between S and P is of the van der Waals type and taking into account the action of the only one perturbing atom, Kuhn [2] obtains an expression representing the intensity distribution in the long wave wing of the broadened line. His measurements on the mercury resonance line 2537 Å as well as those of Minkowski on the sodium D lines broadeend by argon [3] have given a good agreement of the observed intensity distribution with the $(\nu_0 - \nu)^{-3/2}$ law. However, this is not the case with the distribution of intensity in the short wave wing, which shows the departure also from the form predicted by the L. L. W. theory.

The general statistical theory founded on these assumptions was developed by Margenau [4]. The purely quantum mechanical treatment of the pressure broadening problem was developed since 1937 in several papers by Jabłoński [5] — [7]. Emphasizing the close analogy of the problem of pressure broadening of spectral lines to the production of molecular spectra (in both cases the intensity distribution is due to the relative movements of nuclei) and considering all the foreign gas atoms P and one atom S as a giant $N + 1$ atomic molecule, Jabłoński deduces a general formula for the intensity distribution in a broadened spectral line. He shows that with certain asymptotic approximations this distribution goes into that of Kuhn. On the contrary, in the case of the asymptotic conditions corresponding to the region of the validity of the L. L. W. theory, all attempts to give a quantum mechanical foundation to the Lorentz line shape failed [8], [9].

For all these reasons, a more systematic investigation of the influence of the number of collisions per second on the form of the spectral line seemed to be of interest.

Such investigations performed by Horodniczy and Jabłoński for the Hg resonance line in the mixture of Hg + He and Hg + Ar [10] showed that for 300°K and 1300°K the influence of the temperature (that is of

the number of collisions per second) is much less pronounced than it was to be expected according to L. L.W. theories. They pointed out the not too great precision of their measurements and suggested that carrying out such measurements in a more systematic way may be useful.

Experimental

As a light source for the continuous background in the absorption experiment an "end on" discharge hydrogen tube was used. The parallel light beam crossed the absorption silica tube 1 m in length and 30 mm in diameter containing mercury vapour and argon. This tube was placed inside the ceramic tube of an electric furnace (150 cm in length) closed with asbestos discs with silica windows. The temperature at the ends and at the centre of the absorption tube was measured by means of thermocouples. A great quartz autocollimation Hilger E478 spectrograph with dispersion ca 2,7 Å/mm at 2537 Å was used. Agfa "Spectral Blau-Rapid" plates have been found suitable as negative material.

The absorption tube prepared by a standard treatment by heating *sub vacuo*, was filled with spectroscopically pure argon (pressure at 20°C p_{Ar} = 300.4 mm Hg, which corresponds to 1×10^{19} at/ccm.) and sealed off. No special filling with mercury was provided. The mercury is always present in the vacuum apparatus including a mercury diffusion pump (if not frozen out with liquid air) in a quantity which is sufficient to ensure the total absorption in the centre of the resonance line 2537 Å.

By such a procedure of filling the concentration of the mercury vapour in the tube was constant and equal to about 4.2×10^{14} at/ccm. in a range of temperatures higher than 325°K.

The stability of the light source was controlled by means of a photocell. The control of the furnace temperature was assured by a three-phase autotransformer supplying the power to the furnace.

In every measure series the absorption line was photographed on the same plate at 7 different furnace temperatures from about 300°K to 1250°K (the values of other experimental parameters being kept constant). The thermoelectric Moll self-recording microphotometer was used (with 100-fold magnification) for evaluation of the blackening curves. The shape of the examined line and the value of the total absorption being conserved for the different temperatures, it was not necessary to bring the photometric marks and to recalculate the blackening distribution into true intensity distribution. The photometric curves were photographically magnified and the area enclosed by the absorption line was measured by means of an Amsler planimeter.

Results

The results of all measurements show that within the indicated limits of temperature (320°K to 1250°K) the value of the total absorption remains unchanged. The ratio of the area enclosed by the absorption line in the high temperature to that in the temperature of about 400°K remains constant and equal to unity (Fig. 1), the deviation of the mean value for each temperature point (taken from twelve plates) did not exceed 1 ppc (the results of the single measurements being spread out within the limits — 5 ppc to + 5 ppc).

This result agrees well with those of Horodniczy and Jabłoński [10] and is in contradiction to the predictions of the Lorentz theory (40 ppc in increase) as well to that of the Lenz-Weisskopff theory (23 ppc in increase).

To ascertain that the mercury vapour always present in physical laboratories shows no significant absorption (in such a case the observed absorption line should be broadened by air at 1 atm. pressure) some control spectra were taken without the absorption tube. The result was

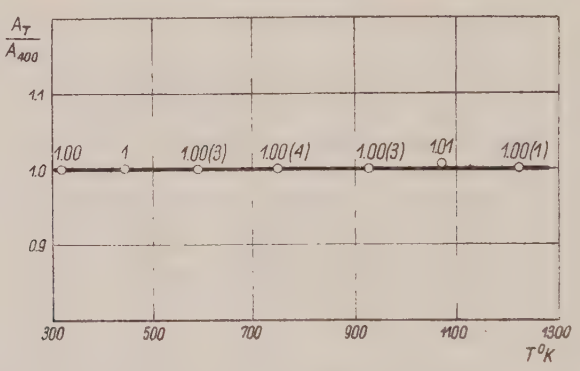


Fig. 1. The ratio the total absorption at high temperature T and that at 400° K *vs.* temperature

negative. The very weak absorption mercury resonance line observed at first completely disappeared when adequate ventilation was installed in the laboratory.

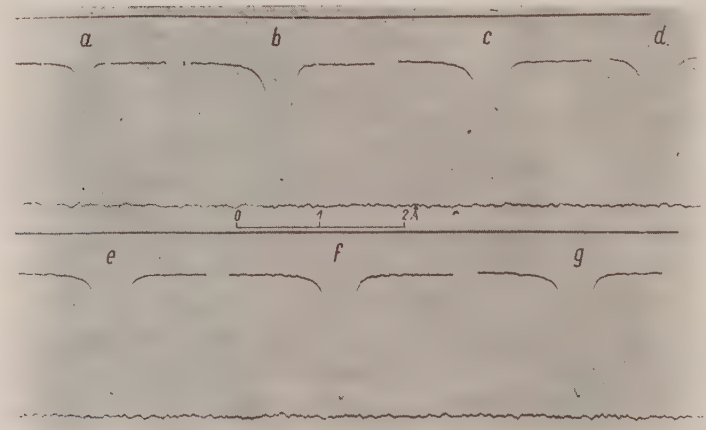


Fig. 2. Microphotometer record of the absorption line at seven different temperatures: a) 293°K b) 440°k c) 590°K d) 750°K e) 920°K f) 1070°K g) 1220°K

The almost constant shape of a line may be seen in Fig. 2 showing microphotograms for seven different temperatures (the first small curve corresponds to room temperature. when the mercury vapour pressure does not reach its final value).

Nevertheless, careful measurements reveal a slight diminution in the asymmetry of the line, when increasing the temperature. The same effect is reported in [10].

The author is indebted to Professor A. Jabłoński for many helpful discussions and to Mr J. Rogaczewski M. Sc. for his kind assistance in the measurements.

INSTITUTE OF EXPERIMENTAL PHYSICS, UNIVERSITY, WARSAW
(INSTYTUT FIZYKI DOŚWIADCZALNEJ, UNIWERSYTET, WARSZAWA)

REFERENCES:

[1] S. Robin et S. Robin, Journ. de Phys. et Rad., **17** (1956), 143.
[2] H. Kuhn, Proc. Roy. Soc., A **158** (1937), 212.
[3] R. Minkowski, Zs. f. Phys., **93** (1935), 731.
[4] H. Margenau, Phys. Rev., **48** (1935), 755.
[5] A. Jabłoński, Acta Phys. Pol., **6** (1937), 371; **7** (1938) 196.
[6] — , Physica, **8** (1940), 541.
[7] — , Phys. Rev., **68** (1945), 78.
[8] H. M. Foley, Phys. Rev., **69** (1946), 616.
[9] A. Jabłoński, Phys. Rev., **73** (1948), 258.
[10] H. Horodniczy, A. Jabłoński, Nature, **142** (1938), 1122; **144** (1939), 594.

БЮЛЛЕТЕНЬ
ПОЛЬСКОЙ АКАДЕМИИ НАУК

СЕРИЯ МАТЕМАТИЧЕСКИХ, АСТРОНОМИЧЕСКИХ
И ФИЗИЧЕСКИХ НАУК

1960

ТОМ VIII Резюме статей ВЫПУСК 2

Авторы дают решение одной проблемы Банаха; доказывается существование нерефлексивного пространства, неизоморфного со своим картезианским квадратом.

Пусть X сепарабельное пространство Банаха и пусть $\delta(X) = \dim_l(X^{**}, \hat{X}) = 1$ ($\hat{X}$ обозначает канонический образ пространства X во втором сопряженном пространстве X^{**}).

ТЕОРЕМА 1. *Пространства X и $X^2 = X \times X$ неизоморфны; сверх того $\dim_l X < \dim_l X^2$.*

ТЕОРЕМА 2. *Если пространство Y слабо полно (или если Y^* слабо полно), то пространства $X \times Y$ и $(X \times Y)^2$ неизоформны.*

Пусть $X = C(\Gamma_\alpha)$ пространство Банаха вещественных функций, определенных на множестве трансфинитных чисел $\Gamma_\alpha = \{\beta : \beta \leqslant \alpha\}$; Γ_α бикомпактно относительно топологии упорядоченности. Пусть ω_1 наименьшее трансфинитное число, которого мощность $\aleph_1$. Рассмотрим следующий инвариант изоморфизмов: множество X_s всех функционалов x^{**} принадлежащих второму сопряженному пространству X^{**} пространства X, для которых множество $\{x^* : x^{**}(x^*) = 0\}$ последовательно замкнуто относительно слабой топологии $\sigma(X^*, X)$.

ТЕОРЕМА 1. *Если $X = C(\Gamma_{\omega_1})$, то элементы множества X_s являются линейными комбинациями канонических образов $\hat{x}^{**}(x^*) = x^*(x)$ элементов $x \in X$ и одного фиксированного функционала $x_0^{**} \in X^{**}$.*

ТЕОРЕМА 2. *Пространство $C(\Gamma_{\omega_1})$ неизоморфно со своим картезианским квадратом $C(\Gamma_{\omega_1 \cdot 2})$.*

Произведены пробы оценки эффекта экзальтации рефракции в некоторых молекулах с сопряженными связями. Полная рефракция представлена в виде суммы двух частей: аддитивной рефракции скелета молекулы и рефракции электронов π. Эта последняя рефракция была вычислена по методу FEMO, модифицированному путем введения периодического потенциала.

Полученные результаты свидетельствуют о том, что такое разделение вполне обосновано. Констатировано, что при применении правильных волновых функций для электронов π можно было бы получить результаты численно согласующиеся с экспериментом.

В настоящей работе пересчитаны матричные элементы энергии для структуры белого олова.

Приводятся таблицы, учитывая, прежде всего, апроксимацию ближайших соседей первого и второго ряда, а затем также принимая двухцентровую апроксимацию.

Показано, что интерпретация двух известных ([1], [2]), „аномальных" случаев распада $K^+ \to \pi^+ +$ нейтр. как $K^+ \to \pi^+ + \bar{\nu} + \nu$ при предположении

a) универсальности V-A связи для заряженных и нейтральных фермионных токов,

b) изоспинорного несохраняющего странность тока сильно взаимодействующих частиц

противоречит эксперименту.

На основании ранее полученного решения [1], исследовался механизм модуляции электронного пучка, развёртывающего поверхность тонкого полупроводящего слоя на потенциальном рельефе этого слоя. Установлено, что в зависимости от угла наклона характеристики вторичной эмиссии тонкого слоя возможны два вида модуляций, отличающиеся полярностью выходного сигнала.

Рассчитаны и построены графики для чувствительности, инерционности и величины точка, протекающего через тонкий слой, в зависимости от сопротивления слоя.

Получена зависимость тока, протекающего через тонкий слой, от ёмкости слоя.

Наблюдался совместный рост концентрации и эффективной массы электронов, определенной из измерений термо-эдс. в InSb. Этот рост объясняется непараболической структурой зоны проводимости (согласно дисперсионной формуле Кана [2]). Теоретическая кривая, обоснованная концепцией обобщенной эффективной массы, в случае непараболичности $E(\vec{k})$, находится в довольно хорошем согласии с экспериментом.

Анализ рассеяния электронов в InSb-n проводится с учетом непараболической структуры зоны. Оказывается, что концепция обобщенной эффективной массы и принятие дисперсионной формулы Кана для зоны проводимости [3] позволяет значительно приблизить теоретическое значение подвижности, обоснованной рассеянием на акустических фононах μ_{ph} к экспериментальному значению. При этом объясняется также зависимость μ_{ph} от концентрации электронов и ход температурной зависимости μ_{ph} для разных концентраций.

Дается описание магнитного фильтра, применение которого дает возможность увеличить однородность магнитного поля в электромагните до значения 10^{-7} в объеме 100 $мм^3$. Этот фильтр был применен в электромагните для ядерного резонанса с большой разрешающей силой. Наблюдалось расщепление линии этилового спирта вследствие воздействий спин-спин.

Были проведены измерения эффективных сечений потенциального рассеяния для элементов со средним атомным весом следующих элементов (Fe, Co, Ni, Cu, Zn, Se).

Результаты не согласуются с оптической моделью для сферических ядер и позволяют предполагать, что в области $30 < A < 90$ имеются также значительные деформации ядер.

TABLE DES MATIERES

Cena zł 20.—